Please show work or give reasoning for every answer. I need some evidence that you understand the topics. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your calculator, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed formula, please write down the formula that you are using.

1. Find the equation of the line which is tangent to the graph of \( f(x) = 2 \ln(x) \) at the point \((x, y) = (1, 0)\).

   (Hint: First find the slope of the tangent line.)

2. Silly Sally can’t remember which trig. derivative has the minus sign:
   "is it \( \frac{d}{dx} \sin(x) = -\cos(x) \) or \( \frac{d}{dx} \cos(x) = -\sin(x) \) ?"

   Assuming that she knows what the graphs of \( \sin(x) \) and \( \cos(x) \) look like, explain how to use the graphs to decide which formula has the minus sign.
3. If $a$ and $b$ are POSITIVE CONSTANTS and

$$g(x) = a(1 - e^{-bx}),$$

show that

(a) $g(x)$ is everywhere increasing

(b) $g(x)$ is everywhere concave down

4. Consider the function $f(x)$ graphed below.

(In responding to the following, estimate $x$-values from the graph.)

(a) Identify the interval(s) on which $f(x)$ is concave up, if any.

(b) Identify any critical points of $f$
    (you need only give the $x$ value(s)).

(c) Identify any inflection points of $f$
    (you need only give the $x$ value(s)).

(d) Identify the interval(s) on which $f'(x) < 0$, if any.
5. Sketch the graph of a function for which \( f'(x) > 0 \) but \( f''(x) < 0 \), or explain why no such function exists.

6. Suppose you are planning a rectangular garden next to your garage, which is 30 feet wide. You have 70 feet of decorative fence to go around the other three sides, as show in the diagram.

(a) Find a formula for \( A(x) \), the area enclosed as a function of \( x \), the width of the garden.

(b) What are the “endpoints” of the function \( A(x) \)?
7. The functions \( r \) and \( s \) and their derivatives satisfy:

\[
\begin{align*}
    r(2) &= 4 & s(2) &= 1 \\
    r(3) &= 2 & s(3) &= 3 \\
    r(4) &= 3 & s(4) &= 5 \\
    r'(2) &= -2 & s'(2) &= 2 \\
    r'(3) &= 1 & s'(3) &= 4 \\
    r'(4) &= 0 & s'(4) &= 0
\end{align*}
\]

Compute the following, or state what additional information you would need to be able to do so.

(a) \( H(2) \), where \( H(x) = s(r(x)) \)

(b) \( H'(2) \), where \( H(x) = s(r(x)) \)

(c) \( G'(2) \), where \( G(x) = r(x) \ast s(x) \)

(d) \( p'(2) \), where \( p(x) = \sqrt{r(x)} \)

(e) \( q'(2) \), where \( q(x) = r(x^2) \)
8. Use derivatives to find the absolute maximum and minimum of the function

\[ f(x) = 35x - \frac{1}{2}x^2; \quad 0 \leq x \leq 30 \]

on the interval \( 0 \leq x \leq 30 \). (Make it clear how you use information from the derivative to draw your conclusion. Do not use your graphing utility (except may be to check your answer).)

9. Consider the graph of the equation

\[ x^2 - 6y^3 = xy. \]

Find the slope of the line tangent to this curve at the point \((x, y) = (3, 1)\).
For these questions, show how to use derivatives to obtain the exact answer.

10. (a) Where is the function \( f(x) = x^3 - 6x + 5 \) increasing?

(b) Where is the function \( f(x) = x^3 - 6x + 5 \) concave up?

(c) Where does the function \( f(x) = x^3 - 6x + 5 \) have a local maximum?

(d) Where does the function \( f(x) = x^3 - 6x + 5 \) have an inflection point?