Chapter 1 Review

1. What is a function?
2. How do you recognize the graph of a function?
3. What does it mean for a function to be invertible?
4. How do you recognize that a function is invertible by its graph?
5. Give an example of a function which is not invertible.
6. Suppose a function \( f \) tells you the number of whatchamacallits as a function of doohickeys. What does \( f^{-1}(6) = 2 \) mean?
7. How do you recognize a linear function from a table? From a graph? From an equation?
8. Given two points on a line, how do you find the equation of the line?
9. How do you recognize an exponential function from a table? From a graph? From an equation?
10. Given two points on an exponential growth curve, how do you find the equation of the curve?
11. What are two different ways to write the equation of an exponential function? How do you switch from one form of equation to the other?
12. How can you tell the difference between exponential growth and exponential decay from a graph? From an equation?
13. What kinds of functions have a characteristic doubling time? ... a characteristic half-life? ... a constant slope?
14. If a function is “increasing”, what does that say about its graph? ... about a table of values for this function?
15. If a function is “concave down”, what does that say about its graph? ... about a table of values for this function?
16. Is every function either even or odd?
17. Is every function either increasing or decreasing?
18. Give an example of the graph of a function which is both concave up and increasing.
19. Give an example of the graph of a function which is both concave down and increasing.
20. Give an example of the graph of a function which is both concave down and decreasing.
21. Give an example of the graph of a function which is both concave up and decreasing.
22. If a function is linear, what can you say about its inverse?
23. If a function is exponential, what can you say about its inverse?
24. If \( P(t) = P_0a^t \), how can we tell if \( P \) represents growth or decay?
25. If \( P(t) = P_0e^{kt} \), how can we tell if \( P \) represents growth or decay?
26. What is the domain of an exponential function? What is its range? What kind of asymptote does it have?
27. Is there a formula for rewriting “\( ax^b \)” in another form? If so, what is it? What about “\( x^n \)” or “\( a^n \)”?
28. What is the domain of a log function? What is its range? What kind of asymptote does it have?
29. Is there a formula for rewriting “\( \ln(xy) \)” in another form? If so, what is it? What about “\( \ln((x+y)) \)” or “\( \ln(x) + \ln(y) \)” or “\( \ln\frac{n}{n} \)” or “\( \frac{\ln A}{\ln B} \)”?
30. What is the domain of a linear function? What is its range?
31. Which of the following functions will be dominant (largest) as \( x \rightarrow \infty \): \( kx^3, 10^{10}x^2, a^x, (\frac{1}{a})^x, \sqrt{2x+2}, \sin(e^x), \arctan(x), e^{kx}, e^{-kx}, \frac{C}{x^2}, x^3, x^{-3}, x^{1/3}, x^{-1/3}, \ln(x), 3 \ln(x), \) or \( \frac{2x+2}{x^2} \)? (Assume everything except \( x \) is a positive constant, and \( a > 1 \).)
32. How can you tell from an equation that a rational function has a vertical asymptote?
33. What are the domains and ranges of the basic trigonometric functions? Which ones have asymptotes, and how do you know where the asymptotes are?
34. What is the domain of the arcsin function? What is the range of the arcsin function? Why is the range of arcsin restricted? (What about the arccos?)
35. What is the domain of the arctan function? What is the range of the arctan function?
36. How do you recognize the amplitude and period of a sine (or cosine) curve from its graph? From an equation?
37. Suppose you have a graph of \( f(x) \). Describe the graphs of \( y = f(3x), y = 3f(x), y = f(x+3), y = f(-x), y = f^{-1}(x), y = -f(x), y = f(x) + 3, y = f(3x) + 3 \).
38. What is a “radian”? Why is this a convenient unit when talking about arc length on a circle?
39. What is the period of the function \( y = \sin(x) \)?
40. What is the period of the function \( y = \tan(x) \)?
41. Describe what is meant when we say that a graph is “symmetric about the origin.” (What about “symmetric about the \( y \)-axis”?)
42. Describe what is meant when we say that a graph has “even symmetry,” ... what about an “odd” function?
43. How do you write an equation which says that one quantity is proportional to another?
44. Suppose \( f(x) \) is a decreasing function of \( x \). What happens to \( f \) when we increase \( x \)? What happens when we decrease \( x \)?

45. What is a “half-life”?

46. What is the “domain” of a function? What is the “range”?

47. Which polynomials have even symmetry? Which polynomials have odd symmetry? Are there polynomials which have neither?

48. From an equation, how can you tell where the graph will cross the \( x \)-axis? The \( y \)-axis?

49. If the graph of a polynomial in \( x \) crosses the \( x \)-axis at a point, what can you say about the polynomial and its factors?

50. What is meant by “the degree of a polynomial”?

51. If \( P(x) \) is a polynomial in \( x \) of degree 4, what does this say about the equation for \( P(x) \)? What does this tell us about the graph of \( y = P(x) \)? (How does it behave as \( x \to \infty \)? How 'bout \( x \to -\infty \)? How many times might it change direction, from increasing to decreasing or vice-versa?) What about polynomials with other degrees?

52. What is the difference between an exponential function and a power function?

53. How can you find \( f(g(x)) \), given a table of values for \( f(x) \) and \( g(x) \)? What if you are given formulas for \( f(x) \) and \( g(x) \)? What if you are only given graphs of \( f(x) \) and \( g(x) \)?

54. If \( g(x) \) is the inverse function of \( f(x) \), what is \( f(g(x)) \)?

How is each item in Column G related to something from Column S?

<table>
<thead>
<tr>
<th>Column G</th>
<th>Column S</th>
</tr>
</thead>
<tbody>
<tr>
<td>• the amplitude of a sinusoidal graph</td>
<td>• the function ( f(x) = mx + b ) (( m ) and ( b ) are constants)</td>
</tr>
<tr>
<td>• the number of “turning points” on a graph (where the graph goes from increasing to decreasing or vice-versa)</td>
<td>• the factor ( a ) in front of ( x ) in ( f(x) = c\sin(ax) + b )</td>
</tr>
<tr>
<td>• one graph looks like the other reflected about the line ( y = x ).</td>
<td>• the factor ( c ) in ( f(x) = c\sin(ax) + b )</td>
</tr>
<tr>
<td>• the slope of a line</td>
<td>• the constant ( b ) in ( f(x) = c\sin(ax) + b )</td>
</tr>
<tr>
<td>• the period of a sinusoidal graph</td>
<td>• the leading coefficient and/or the leading term of a polynomial</td>
</tr>
<tr>
<td>• If you fold the graph along the ( y )-axis, it matches up with itself</td>
<td>• the function ( f(t) = ca^t ) (( c ) and ( a ) are constants)</td>
</tr>
<tr>
<td>• if you rotate the graph by 180° about the origin, it doesn’t change</td>
<td>• the function ( f(t) = ce^{kt} ) (( c ) and ( k ) are constants)</td>
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<tr>
<td></td>
<td>• odd symmetry: ( f(-x) = -f(x) )</td>
</tr>
<tr>
<td>• the ( x ) intercept of an exponential function</td>
<td>• even symmetry: ( f(-x) = f(x) )</td>
</tr>
<tr>
<td>• the ( y ) intercept of an exponential function</td>
<td>• inverse functions</td>
</tr>
<tr>
<td>• the ( x ) intercept of a logarithmic function</td>
<td>• the function ( f(x) = a\ln(bx + c) ) (( a, b, ) and ( c ) are constants)</td>
</tr>
<tr>
<td>• the ( y ) intercept of a logarithmic function</td>
<td>• the degree of a polynomial</td>
</tr>
<tr>
<td>• vertical asymptote</td>
<td>• the factors of a polynomial function</td>
</tr>
<tr>
<td>• horizontal asymptote</td>
<td>• the factors of the denominator of a rational function</td>
</tr>
<tr>
<td>• the ( x ) intercept(s)</td>
<td>• the factors of the numerator of a rational function</td>
</tr>
<tr>
<td>• the ( y )-intercept of a line</td>
<td>• the end behavior of a graph (i.e., do the tails go to ( +\infty ) or ( -\infty )?)</td>
</tr>
<tr>
<td>• the graph of ( f ) increases exponentially</td>
<td>• the graph of ( f ) shows exponential decay</td>
</tr>
<tr>
<td></td>
<td>• the vertical shift of a sinusoidal graph</td>
</tr>
</tbody>
</table>
Study Suggestions

Besides thinking about the questions on the review sheet, here are some other ways to study for the test:

- Skim through each section of the book and identify the BIG IDEA of that section. Try to summarize it in one or two sentences.

- After you’ve identified the big idea in each section, check that you can express each idea graphically, symbolically, and numerically. Know how these three interpretations are related.

- Re-read your worksheets. For each part, ask yourself “What was the point of this problem? What was I supposed to see/learn?” or “How are all these things connected?”

- Work through the scoop on reserve in the library. Treat it like a real test and force yourself to think through the questions.

- Skim through the lab projects. What ideas were presented? Which concepts are illustrated in the graphs?

- Review your quizzes on this material.

- Review your homework on this material. Don’t try to memorize procedures, but try to summarize the main skills and relationships that you used to find your solutions.

- Re-read the examples in the book. After going through each example, try to summarize the procedure or idea in a few sentences. What is it an example of?

- Make a list of formulas you will need, and memorize them and/or program them in your calculator. Be sure you know how and when to use them!

- Read through the review problems in the book and decide how you would approach each one. Concentrate on setting up the problems: how do you know what the problem is asking for, and what mathematical ideas/tools are needed to solve it?