Final Exam – Part I  
MA1160, Spring ’02

NO CALCULATORS ALLOWED  
on Part I

Please show work or give reasoning for every answer. We need some evidence that you understand the topics. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

1. Find each of the following.

(a) \[
\frac{d}{dx} \left( x^3 + 3^x + \frac{\pi}{x^2} + \pi \right)
\]

(b) \[
\int \left( x^3 + 3^x + \frac{\pi}{x^2} + \pi \right) \, dx
\]

(c) \[
\frac{dy}{dx} \text{ where } y = x \ln(x)
\]

(d) \[
f'(x) \text{ where } f(x) = \frac{e^{3x}}{\sin(x)}
\]
(e) \( \frac{dh}{dt} \) where \( h = \arctan(\sqrt{t}) \)

(f) \( \int_{1}^{3} \frac{x^2 + 3}{x} \, dx \)

(g) \( \frac{dy}{dx} \), where \( x \) and \( y \) are on the graph of \( x^3 + y^3 = 6xy \)
2. For the function \( f(x) = x^3 - 6x^2 + 6 \),

(a) Find the critical points of the function.

\[
\quad /3 \text{ points}
\]

(b) Classify each critical point as a local maximum, a local minimum, or neither.

\[
\quad /3 \text{ points}
\]

(c) Find the inflection points of the function.

\[
\quad /3 \text{ points}
\]

(d) Find the global maximum and minimum of \( f(x) \) on the interval \(-1 \leq x \leq 1\).

\[
\quad /3 \text{ points}
\]
Please show work or give reasoning for every answer. We need some evidence that you understand the topics. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your calculator, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed formula, please write down the formula that you are using.

3. Give an example of an exponential function and an example of a power function and the derivative of each.

<table>
<thead>
<tr>
<th></th>
<th>function</th>
<th>derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponential</td>
<td></td>
<td></td>
</tr>
<tr>
<td>power</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

/4 points

4. A function is exponential. What type of function is its inverse?

/2 points

5. A cup of coffee comes out of the microwave at 180°F and into a room where the temperature is 70°F.

Which one of these formulas best describes the temperature of the coffee $t$ minutes after being removed from the microwave? (Circle only one.)

(a) $T(t) = 180 - 70e^{-t/2}$
(b) $T(t) = 70 + 110e^{-t/2}$
(c) $T(t) = 250 - 70e^{-t/2}$

/2 points
6. Some values of two functions are given in the following table. For each of the questions, there may be several correct answers or there may be none. If none of the given functions has the indicated property, write “NONE.”

<table>
<thead>
<tr>
<th></th>
<th>( g(x) )</th>
<th>( h(x) )</th>
<th>( k(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>6.5</td>
<td>700</td>
<td>1875</td>
</tr>
<tr>
<td>3.2</td>
<td>6.0</td>
<td>740</td>
<td>1500</td>
</tr>
<tr>
<td>3.3</td>
<td>5.5</td>
<td>770</td>
<td>1200</td>
</tr>
<tr>
<td>3.4</td>
<td>5.0</td>
<td>790</td>
<td>960</td>
</tr>
<tr>
<td>3.5</td>
<td>4.5</td>
<td>800</td>
<td>768</td>
</tr>
</tbody>
</table>

(a) One of these functions is a linear function. Identify the linear function and find its slope.

(b) Which of these functions could represent exponential decay (if any)?
   (How can you tell?)

(c) Which of the functions is increasing?
   (How can you tell?)

(d) Estimate the value of \( h'(3.3) \).
7. Consider the sinusoidal function, \( f(x) \), graphed below.

\[ y = f(x) \]

(a) What is the period of \( f \)?

(b) What is the amplitude of \( f \)?

(c) Is \( f(x) \) an even function, an odd function, neither, or both?

(d) Give a possible formula for \( f(x) \).

8. Sketch the graph of a function \( f(x) \) for which \( f'(x) > 0 \) and \( f'(x) \) is decreasing, or explain why no such function exists.

9. A yam has just been taken out of the oven and is cooling off before being eaten. The temperature, \( T \), of the yam (measured in degrees Fahrenheit) is a function of how long it has been out of the oven, \( t \) (measured in minutes). Thus we have \( T = f(t) \).

(a) Is \( f'(t) \) positive or negative? Why?

(b) What are the units for \( f'(t) \)?
10. Suppose the position, \( s(t) \), of a car on the Garden State Parkway is shown on the graph below, where \( s \) is measured in miles from exit 63 and \( t \) is measured in hours.

(a) What is the average velocity of the car over the first three hours? Include units.

(b) What is the instantaneous velocity of the car at the end of the second hour (\( t = 2 \))? Include units.

11. Let \( f(x) \) and \( g(x) \) be two functions with values given in the tables below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( x )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-2</td>
<td>3</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Use the information in the table to find \( m'(1) \), where \( m(x) = f(g(x)) \).
12. Find the equation of the line tangent to the graph of \( y = 3 + 2e^x \) at the point \((0, 5)\).

\[
\text{\underline{---/4 points}}
\]

13. Answer the following questions about \( f(x) \) based on the graph of its \textit{derivative} below.

\[
y = f'(x)
\]

\[
\leftarrow y = \frac{df}{dx}
\]

(a) Over what interval(s) is \( f \) increasing?

\[
\text{---/2 points}
\]

(b) List the critical points of \( f \), if any.

\[
\text{---/3 points}
\]

(c) List the inflection points of \( f \), if any.

\[
\text{---/2 points}
\]

(Reminder: the graph above is the \textit{derivative} of \( f \), while the questions ask about \( f \) itself)
14. Suppose you are instructed to draw a rectangle with its base on the $x$-axis and two vertices on the parabola $y = 8 - x^2$, as shown in the figure below.

(a) Find an equation for the area of the rectangle as a function of $x$.

(b) Use derivatives to find the dimensions of the rectangle with the largest possible area. (Show how you use derivatives to reach your conclusion.)
15. Find the shaded area.

16. A jet plane accelerates from a standing stop at a constant 10 feet/sec². It needs to be going 300 feet/second to take off. How far down the runway does it go before taking off?
17. (a) Find \( \int_{-1}^{3} f(x) \, dx \), where the graph of \( f(x) \) is shown.

(b) If \( F(2) = 13 \) and \( \int_{2}^{4} F'(x) \, dx = 10 \), find \( F(4) \).

18. A car travels down a straight road without stopping. The driver sets a timer to zero as she passes mile marker 25. As she drives along, she keeps the following record:

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Maximum Speed (miles per hour)</th>
<th>Minimum Speed (miles per hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st}) hour</td>
<td>72</td>
<td>35</td>
</tr>
<tr>
<td>2(^{nd}) hour</td>
<td>81</td>
<td>46</td>
</tr>
<tr>
<td>3(^{rd}) hour</td>
<td>55</td>
<td>25</td>
</tr>
<tr>
<td>4(^{th}) hour</td>
<td>70</td>
<td>60</td>
</tr>
<tr>
<td>5(^{th}) hour</td>
<td>72</td>
<td>39</td>
</tr>
</tbody>
</table>

Which of these mile markers is it certain that she passed during the first 5 hours? Show how to come to your conclusion.

200 215 225 240 320
19. The amount of space (in cubic feet) purchased by Triple A’s customers is modeled by

\[ f(x) = 50 + 120x - 3x^2, \quad 0 \leq x \leq 30, \]

where \( x \) is in days.

(a) What was the average amount stored by Triple A over the 30-day period?

(b) What are the units of \( \int_0^{30} f(x) \, dx \)?

\[ \underline{4 \text{ points}} \]

\[ \underline{2 \text{ points}} \]