Section 16.5: Integrals in Cylindrical Coordinates

Sketch the region of integration and use cylindrical coordinates to evaluate the integral

$$\int_W f dV,$$

where f and W are given by

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a. $f(x, y, z) = z\sqrt{x^2 + y^2}$ and W is the hemisphere of radius 2 described by $x^2 + y^2 + z^2 \leq 4, \quad z \geq 0.$

(Answer: $\frac{64\pi}{15}$)

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b. $f(x, y, z) = \sqrt{x^2 + y^2}$ and W is bounded by $z = \sqrt{x^2 + y^2}$, $z = 0$, and $x^2 + y^2 = 1$.

(Answer: $\frac{\pi}{2}$)

Section 16.5: Integrals in Spherical Coordinates

Sketch the region of integration and use spherical coordinates to evaluate the integral

$$\int_W f dV,$$

where f and W are given by

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1. $f(x, y, z) = z^2$
   W is the quarter sphere of radius 1 described by $x^2 + y^2 + z^2 \leq 1, \quad y \geq 0, \quad z \geq 0.$

(Answer: $\frac{\pi}{15}$)

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2. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$
   W is the region above the cone $z = -\sqrt{3x^2 + 3y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

(Answer: $8\pi + 4\pi\sqrt{3}$)