Section 16.5: Integrals in Cylindrical Coordinates

Sketch the region of integration and use cylindrical coordinates to evaluate the integral

$$ \int_W f \, dV $$

where $f$ and $W$ are given by

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a. $f(x, y, z) = z \sqrt{x^2 + y^2}$ and $W$ is the hemisphere of radius 2 described by $x^2 + y^2 + z^2 \leq 4, \quad z \geq 0.$

(Answer: $\frac{64\pi}{15}$)

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b. $f(x, y, z) = \sqrt{x^2 + y^2}$ and $W$ is bounded by $z = \sqrt{x^2 + y^2}, \quad z = 0,$ and $x^2 + y^2 = 1.$

(Answer: $\frac{\pi}{2}$)
Section 16.5: Integrals in Spherical Coordinates

Sketch the region of integration and use spherical coordinates to evaluate the integral

\[ \int_W f dV, \]

where \( f \) and \( W \) are given by

1. \( f(x, y, z) = z^2 \)
   
   \( W \) is the quarter sphere of radius 1 described by \( x^2 + y^2 + z^2 \leq 1, \ y \geq 0, \ z \geq 0. \)

   (Answer: \( \frac{\pi}{15} \))

2. \( f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \)
   
   \( W \) is the region above the cone \( z = -\sqrt{3x^2 + 3y^2} \) and inside the sphere \( x^2 + y^2 + z^2 = 4. \)

   (Answer: \( 8\pi + 4\pi \sqrt{3} \))

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