Please show work or give reasoning for every answer. I need some evidence that you understand the topics. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your calculator, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed formula, please write down the formula that you are using.

1. Suppose that after 5 years, only 60% of a radioactive substance is left.

   (a) Fill in the following table, where $P$ is the percentage of the original amount left after $t$ years.

<table>
<thead>
<tr>
<th>$t$ (years)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (percent)</td>
<td>100</td>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   (b) Find a formula for $P(t)$.

2. Give a formula for a rational function which has $x$-intercepts at $x = 2$ and $x = -3$ and a vertical asymptote at $x = 1$. 


3. Suppose $y$ is the temperature in degrees Fahrenheit and $x$ is the temperature in degrees Celsius, which are related through a function $f$:

$$y = f(x), \quad \begin{cases} x : \text{degrees Celsius} \\ y : \text{degrees Fahrenheit} \end{cases}$$

(a) What does “$f(20)$” represent?

(b) What does “$f^{-1}(20)$” represent?

(c) What is the meaning of the equation $f(100) = 212$?

(d) Write an equation (similar to “$f(100) = 212$” in the previous question) involving $f^{-1}$ which means “32 degrees Fahrenheit is 0 degrees Celsius.”

(Note: $f^{-1}$ denotes the inverse function of $f$.)

The temperature in degrees Fahrenheit, $y$, is given by a linear function of $x$, i.e., the graph of $y = f(x)$ is a line.

(e) What are the units of the slope of the line?

(f) Find an equation for $f(x)$, given that $f(0) = 32$ and $f(100) = 212$. 


4. Shown below are graphs of \( y = 2^x \) and \( y = x^3 + 9x^2 - 4 \).

(a) How do you know that there must be another intersection to the right of the viewing window?
Be specific: what property or properties of the graph(s) tell you there must be another intersection?

(b) How do you know that there must be another intersection to the left of the viewing window?
Be specific: what property or properties of the graph(s) tell you there must be another intersection?

5. Show how to use logarithms to solve the following equation by hand:

\[
20 = 100(.7)^{t/10}
\]

Show all steps clearly.
6. The entire graph of a function, \( y = f(x) \),
is shown at right and its endpoints are labeled.

(a) What feature of the graph shows that it is the graph of a function?

(b) What feature of the graph shows that the function has an inverse?

(c) What is the domain of \( f \)?

(d) What is the range of \( f \)?

(e) What is the value of \( f(2) \)?

(f) What is the value of \( f^{-1}(2) \)?

(g) Where is \( f \) increasing? (indicate \( x \) values)

(h) Where is \( f \) concave up? (indicate \( x \) values)
7. Consider the function \( h(x) = 3 + 2\sin(5x) \).

(a) What is its amplitude?

(b) What is its period?

8. Silly Sally printed out the following graph, but forgot to print the Mathematica code that generated them. Now she can’t remember if she graphed \( \sin(x) \) and \( \sin(2x) \), or \( \sin(x) \) and \( \sin(\frac{1}{2}x) \).

Sally understands that the solid line is \( \sin(x) \) because she knows \( \sin(x) \) crosses the \( x \)-axis at \( x = 0 \), \( x = \pi \), and \( x = 2\pi \). Explain how she can figure out whether she graphed \( y = \sin(2x) \) or \( y = \sin(\frac{1}{2}x) \), without resorting to a graphing utility.

9. Suppose the numbers \( a \) and \( b \) satisfy \( b = \sin^{-1}(a) \). Which one represents an angle?
10. The tangent function can be defined in terms of sine and cosine.

(a) What is the formula? \[ \tan(x) = \ldots \]

(b) Why does the tangent function have asymptotes, even though the sine and cosine do not?

11. Suppose an exponential function \( f(x) = e^{kx} \) satisfies \( f(3) = 1 \) and \( f(7) = 6 \).

(a) Is the graph of \( f \) increasing or decreasing? How do you know?

(b) Is the graph of \( f \) concave up or concave down? How do you know?

(c) Sketch a rough graph of \( f \) and label its asymptote(s).