1. Suppose a simple function $f(x)$ has the representation

$$f = \alpha_1 \chi_{E_1} + \alpha_2 \chi_{E_2},$$

where $E_1$ and $E_2$ are NOT disjoint. Write out the (canonical) representation of $f$ in terms of disjoint sets. Show that the value of the integral $\int f$ is the same, regardless of which representation is used to compute it.

2. Suppose $f$ is a real-valued function on $[a, b]$ and $g$ and $h$ are its lower and upper envelopes, respectively. Prove that $g(x_0) = h(x_0)$ if and only if $f$ is continuous at $x_0$. 