1. What is a derivative? (at least two answers)

2. Given a graph, how can you estimate the derivative at a point?

3. Given a formula for a function \( f \), how can you approximate the derivative at a point if you don’t know the formula for the derivative function?

4. How can you approximate the derivative at a point from a table of function values?

5. Given a graph of a function \( f(x) \), how can you sketch a graph of the derivative function \( f'(x) \)?

6. Given the graph of a derivative \( f'(x) \), how can you sketch a graph of the original function \( f(x) \)?

7. What is the difference between “average velocity” and “instantaneous velocity”? How do you compute each, given position \( s \) as a function of time \( t \)?

8. Suppose a function \( f \) is measured in flugels and it is a function of \( x \), measured in xiapets. What are the units of \( \frac{df}{dx} \)?

9. Describe the meaning of “\( g'(4) = 9 \)” in terms of inputs and outputs.

10. What is a difference quotient? How can you use one to approximate the derivative of a function at a point?

11. What is the limit definition of derivative? How is it related to an average rate of change? Why does it involve a limit?

12. How do you use the limit definition to find a formula for the derivative function (for simple functions like power functions or polynomials)?

13. Explain why the following two statements are NOT saying the same thing: (1) “\( f \) is increasing” vs. (2) “the slope of \( f \) is increasing”. What do each of these statements mean?

14. Given a graph of a function \( f \) and a point on the graph \((x, f(x))\), what do the following quantities represent: \( f(x) \)? \( f(x + h) \)? \( f(x + h) - f(x) \)? \( h \)? What do these all have to do with derivatives?

15. If a function \( f(x) \) is increasing, what does that say about \( f'(x) \)? Does it say anything about \( f''(x) \)?

16. Suppose we know that \( f'(x) \) (the derivative of a function \( f \)) is positive and decreasing. What does that say about the graph of \( f(x) \)?

17. Are there functions (graphs) which are not differentiable at a point?

18. Suppose we know that \( f'(x) \) (the derivative of a function \( f \)) is positive and decreasing. What does that say about the graph of \( f'(x) \)? What about \( f''(x) \)?

19. What does the second derivative, \( f''(x) \) tell you about the graph of \( f(x) \)?
20. What does the second derivative, \( f''(x) \) tell you about the graph of \( f'(x) \)?

21. Explain why acceleration is the second derivative of the position function.

22. If the derivative of \( f(x) \) is positive over an interval, what does that tell you about \( f \) itself?

23. If \( f''(x) \) is positive over an interval, what does that tell you about \( f \) itself?

24. Give an example of the graph of a function \( f(x) \) for each of the following scenarios:
   
   - \( f'(x) > 0 \) and \( f''(x) > 0 \)
   - \( f'(x) > 0 \) and \( f''(x) < 0 \)
   - \( f'(x) < 0 \) and \( f''(x) > 0 \)
   - \( f'(x) < 0 \) and \( f''(x) < 0 \)

25. Suppose you know that \( f(500) = 10 \) and \( f'(500) = .2 \). What can you say about the graph of \( f(x) \)? What can you say about \( f(503) \)? What about \( f(499) \)?

26. Given a formula for a function \( f(x) \), how do you find the equation for the line tangent to \( f \) at \( x = 3 \)? What derivative do you need to compute/approximate first?

27. For which kinds of functions do we have formulas for the derivative?

28. How can you tell the difference between a power function and an exponential function? What are the derivative formulas in each case?

29. How do you use the power rule to find the derivative of something like \( f(x) = \sqrt{x} \) or \( f(t) = \sqrt{t} \)?

30. How do you use the power rule to find the derivative of something like \( f(x) = \frac{1}{x} \) or \( f(t) = \frac{t^2+7}{t^4} \)?

31. For which type of function is the derivative proportional to the function itself?

32. What are the product and quotient rules? How do you know if you need to use them?

33. True or False?
   (Why?)
   
   - ? The derivative of a constant is zero.
   - ? The derivative of a constant times a function is zero.
   - ? The derivative of a sum is the sum of the derivatives.
   - ? The derivative of a product is the product of the derivatives.
   - ? If we multiply a function by 2, its derivative is doubled.

34. Write out the product rule in “primes” notation, i.e., \( (fg)' = \ldots \)

35. Write out the product rule in Leibniz notation, i.e., \( \frac{d(fg)}{dx} = \ldots \)

36. Write out the quotient rule in “primes” and Leibniz notations.