Effective moduli of granular and layered composites with piezoelectric constituents

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ABSTRACT

The effective elastic and dielectric moduli of a composite made from piezoelectric materials are examined, with particular emphasis on applications to unpoled piezoelectric ceramics and layered materials.

Explicit formulae for the effective moduli and coupling of a layered material are derived. A self-consistent estimate of the moduli of an isotropic polycrystal is obtained through an effective medium approximation (EMA), which takes into account the interaction between each individual grain and the surrounding composite. This estimate shows that the grains behave as uncoupled grains with electric and elastic constants modified by the behavior of surrounding grains. A similar effect is also observed in bounds (established via classical variational principles) on the moduli of a statistically isotropic polycrystal. Numerical implementation of the EMA and bounds shows good agreement with data for unpoled barium titanate ceramic. For a general composite with piezoelectric constituents, it is shown that the effective electromechanical coupling can be bounded by the largest coupling factor of the components.

1. INTRODUCTION

Many engineering materials exhibit inhomogeneity on a small scale, either through their polycrystalline nature or because they are composed of mixtures of other materials. Understanding the relationship between the individual small-scale properties and the overall behavior of the material is important in design and processing.

We address the question of macroscopic behavior for materials made from piezoelectric constituents. Generally speaking, the field equations for a given geometry cannot be solved explicitly, so we seek to describe the effective properties through various estimates.

After introducing the necessary ideas and notation in the first section, we find the formulae for the macroscopic moduli in one case where they are calculable: a laminate. Next we describe an approximation scheme in which the electric and elastic fields in each grain are approximated by those of a sphere in an effective medium. These approximate fields lead to approximations of the effective moduli, which are calculated numerically for some polycrystalline materials. In the succeeding section, rigorous bounds on the permittivity and elastic constants are derived and are calculated for the special case of equiaxed polycrystals of Rochelle salt. Finally, we calculate an absolute upper bound on the macroscopic coupling tensor based on the coupling tensors of the constituents.

2. PRELIMINARIES

We shall be concerned with materials which exhibit a linear coupling between electrical and elastic properties. We shall write the linear dependence of the stress τ and electric field E on the strain e and electric displacement D as

$$\tau_{ij} = S_{ijmn} e_{mn} + d_{ij,k} E_k \tag{1}$$

$$D_i = d_{mn,i} e_{mn} + \epsilon_{ik} E_k \tag{2}$$