Improvements on Taylor's Upper Bound for Rigid/Plastic Composites*

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Abstract

A method is described for bounding the effective yield behavior of a mixture composed of anisotropic rigid/plastic materials. It is assumed that the only mechanism for deformation is plastic deformation of individual grains and that the set of stresses causing plastic flow (the "yield surface") is known for each constituent material.

Variational principles are used to generate an upper bound on the effective yield surface from bounds on the elastic moduli of a suitably chosen *linearly* elastic medium having the same geometry as the given material. This work is then compared to that of Taylor and Bishop & Hill, who derived an upper bound for polycrystals deforming by slip. Their results are shown to be recovered when using the elementary arithmetic-mean bound for the linear material. Moreover, it is shown that if one uses more sophisticated bounds on linear properties then the new bound is tighter than that of Taylor and Bishop & Hill. An added advantage of the proposed bounding procedure is the ability to obtain bounds which depend on two-point correlation functions by using the analogous linear bounds.

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1 Introduction

Many engineering materials deform plastically under sufficiently large stress, and the question of precisely how much stress is "sufficient" is an important one in the use and processing of such materials. When the material of interest is actually a *mixture* of materials, then a variety of behavior is possible depending on how these materials are mixed (the microstructure). Describing these possible behaviors often involves establishing bounds on the properties based on given information, which usually includes volume fractions and the properties of the individual constituent materials. The goal of this paper is to enable the calculation of bounds on the effective yield behavior of mixtures, using results from linear elasticity.

The "yield surface" consists of the stresses which are just large enough to cause yield, forming the boundary in stress-space between elastic and plastic deformation. In a homogeneous material under uniform applied stress, the stress throughout the body is uniformly distributed so that the applied stress determines the yield surface. In a mixture, the stress will vary through the material and it is a nontrivial question to ask when any given component within the mixture will reach its point of yield. Perhaps the first to consider this question was Sachs, who proposed approximating the actual stress by a constant stress throughout the material and calculated the effective yield surface under this assumption [1].

Ten years later, G.I. Taylor encountered the same question: how to estimate the effective yield behavior of a mixture? Taylor was interested in describing a polycrystal, which is a mixture of anisotropic crystallites having various orientations [2]. Uncomfortable with the fact that the strains in the Sachs model would be incompatible, Taylor proposed an estimate in which the crystallites would all undergo the same strain (Bishop and Hill also did substantial work on this problem [3, 4]). This estimate proved to be an upper bound on the yield surface, while Sachs' proved to be a lower bound.

For many years, these were the only estimates available for the yield behavior of a mixture. Recently, three independent groups have established new bounds. Ponte Castañeda & deBotton [5] and Michel & Suquet [6] have derived bounds based on homogenization principles for the mixture of von Mises materials. In the present paper, we discuss another bound for general anisotropic mixtures of anistropic materials (including the polycrystal case), which agrees with the other two whenever they all apply (see [7]).

We begin by reviewing the concept of yield and the method for developing an outer envelope of hyperplanes enclosing the effective yield surface of a mixture. Starting from the minimization of energy dissipation, in section (4) we derive our new upper bound which depends on bounding the elastic behavior of a nonhomogeneous linear comparison medium. The following section addresses how our new variational principle allows improvement on the results of Taylor and Bishop & Hill. In the last section, we calculate new yield bounds for mixtures of two materials obeying a von Mises yield criterion.

2 Yield in Homogeneous Rigid/Plastic Materials

The materials which comprise our mixture are assumed to be rigid/plastic, so that stresses below a certain critical value do not cause any deformation. When the magnitude of the stress reaches this critical value, then the material begins to deform plastically ("yield"). If the constituent materials exhibit plastic anisotropy, this critical magnitude of stress will of course depend on the direction of the stress relative to the constituent material.