Lecture: Section 17.3

Vector Fields

A vector field is a function that assigns a vector to each point in space. The gradient of a function $F(x, y)$ produces a vector field. $\vec{\nabla}f(x, y)$ is a vector at each point that points in the direction of maximum increase of $f$.

The figure above shows a gradient vector field superimposed on a contour plot for $f(x, y) = x^2 + y^2$.

The contours are defined by the equation $x^2 + y^2 = k$ and $\vec{\nabla}f = 2x\hat{i} + 2y\hat{j}$.

Velocity Vector Fields – Practical Application

The figures below shows relate to the vortex that forms at the wingtip of an aircraft. An airfoil generates lift due to the velocity differences between the upper and lower surfaces. The camber of the wing causes air to flow faster over the upper surface. Due to Bernoulli’s principle, the pressure on the upper surface is coincidently lower. At the tips of the wings, air tends to flow from the higher pressure surface below to the top of the wing. This flow is rotational, as it flows around the wing tip, and produces a vortex.
Definition of a Vector Field

A vector field in 2 dimensional space is a function whose value at a point \((x, y)\) is in a 2 dimensional vector. In 3 dimensional space, the values are 3 dimensional vectors. For a wingtip vortex, a plot of the vector field might resemble the one shown below.

Sketching Vector Fields

Exercise 17.3.9: Sketch the vector field of \( \vec{F}(x, y) = -y\hat{j} \) in the xy-place.

We see the field only has a \( \hat{j} \) component, that is dependent upon the value of the y-coordinate.

The vectors increase in length as the distance from x-axis increases. Also, the vectors point in the negative y direction.

Finding Formulas of Vector Fields

Exercise 17.3.5
• We see that the formula must have both \( \mathbf{i} \) and \( \mathbf{j} \) components.
• Moreover, the vector field is perpendicular to the \( x \) and \( y \) axes when it crosses.
• The size of the vector increases as the distance from the origin increases.

\[ F(x, y) = -y \mathbf{i} + x \mathbf{j} \] would be consistent with this field. Other answers are, of course, possible.