Lecture: Section 12.1

- Function of two variables
  - There are many instances where the response of a system is dependent upon more than a single variable.
  - In previous calculus courses, when such systems were considered, all but one of the variables was held constant, and single variable calculus techniques were applied.
  - An example of this might be seen in the following problem:
    - Let's say we immersed a heated cylindrical bar in a bath of cold water.
    - We know that, from Newton's Law of Cooling, we could model the temperature of the bar as a function of time.

![Diagram of a heated cylindrical bar in a bath of cold water.](image)

- In the sample analysis of this Problem, the initial temp of the bar is known and the temp of the bath is assumed to remain constant.
- The solution to the problem is of the form:

\[ T = T_{\text{Bath}} + (T_{\text{Initial}}) (e^{kt}) \]

Where \( k \) is related to the heat transfer characteristics of the system and is assumed to be constant in this case.

- What are some factors that we consider that might turn into a multivariable problem?
  1. What if the bar were very thick?
  2. What if the bar was large compared with the volume of the water?
  3. Are heat transfer properties always temperature dependent? What if \( \Delta T_{\text{Initial}} \) were very large?
Here is another common example of a multivariable problem.

- See the weather map on p.604
  - See map isotherms
  - What is needed to determine a temperature at a point?
  - How could this be extended to a three variable problem?

A weather map

Three Dimensional Space

- We will start by extending our 2-D Cartesian coordinate system to 3 dimensions. This coordinate system is often also referred to as rectangular for obvious reasons.
- We will also define the system to be right-handed... which means to that the orientation of the x, y, z axes is specified in a certain way.

Fingers of right hand point in the direction of x-axis. If hand is closed, x will rotate in the direction of y and thumb will point in (+) z direction.
Let’s try an example. If the origin is at the front, right, bottom corner of the room:
1. Define x, y, z axes
2. Find coordinates of center of room
3. Find coordinates of center of room below.

Planes in 3-D Space

Note that in our room example, a plane would be like a wall, a line, e.g., an edge where wall meets floor (i.e. intersection of 2 planes) and a point, e.g., the corner where we set the origin.

To specify a position in/on the following.
- Room – 3 variables needed
- Wall -- 2 variables needed
- Edge -- 1 variable needed
- Corner - 0 variables needed

Examples
1. Which of the following points is in the xy plane? (0,1,1), (1,1,0), (1,1,1)?
2. Which of the following points is closest to the yz plane? (1, 1, 1), (0.5, 1, 3), (3, 1, 0)?

Distance between two points
- In two dimensions we found that the distance between two points was determined as follows:
d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2

d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}

- We could apply the Pythagorean Theorem twice to extend this to 3-D.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \]

- Here are a couple of examples
  - Example 12.1.5
    - P = (1, 2, 1) \ Q = (2, 0, 0)
    - Which is closer to the origin, O = (0, 0, 0)?
      - PO = \sqrt{(1-0)^2 + (2-0)^2 + (1-0)^2} = \sqrt{6}
      - QO = \sqrt{(2-0)^2 + (0-0)^2 + (0-0)^2} = \sqrt{4}
      - So Q is closer to O.
  - Example 12.1.7
    - Find the equation of a sphere of r = 5, centered at origin.
      - We will define a point on the sphere to be (x, y, z)
      - By distance formula:
        \[ \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = r = 5 \]
        or: \[ 25 = x^2 + y^2 + z^2 \]