MA3160
Quiz 9 – Summer 2007
10th August 2007
Due: 14th August 2007
Open Book/Notes

Name: Key

1.) Calculate the line integral of $\vec{F} = -y\vec{i} + x\vec{j}$ along the following paths.

a.) straight line from the origin to the point (2, 3)

$r(t) = t(2\bar{i} + 3\bar{j}) \qquad r'(t) = 2\bar{i} + 3\bar{j}$

$\int_c \vec{F} \cdot d\vec{r} = \int_c \vec{F}(t) \cdot r'(t)\, dt = \int_c (-y\bar{i} + x\bar{j}) \cdot (2\bar{i} + 3\bar{j})\, dt$

$x = 2t \qquad y = 3t \qquad\qquad t = 0 \text{ to } t = 1$

$= \int_c (-6t + 6t)\, dt$

$= \int_0^1 0\, dt = 0$

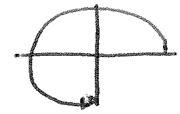
b.) Counterclockwise around the perimeter of a triangle of area 7.

$\text{Curl } \vec{F} = 1 - (-1) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2$

Use Green's Th.

$\int \vec{F} \cdot d\vec{r} = \int_c \text{curl } \vec{F}\, \bar{k} \cdot dA\, \bar{k} = \int \text{curl } F\, dx\, dy$

$= 2\int_c dx\, dy = 2A = 2 \cdot 7 = \boxed{14}$



2.) Compute the line integral of $\vec{G} = (x-y)\vec{i} + (x+y)\vec{j}$ along a counterclockwise path around a circle of radius 2, centered at the origin, starting at (2, 0) and ending at (0, -2).

$$\text{Curl } \vec{G} = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 1 - (-1) = 2$$

$$r(t) = \underset{(x(t))}{2\cos t\, \vec{i}} + \underset{g(t)}{2\sin t\, \vec{j}} \qquad \vec{G} = [\cos t - \sin t]\vec{i} + [\cos t + \sin t]\vec{j}$$

$$r'(t) = 2(-\sin t\, \vec{i} + \cos t\, \vec{j}) \qquad t = 0,\ 3\pi/2$$

$$2 \cdot 2 \int^{3\pi/2} [\cos t - \sin t]\vec{i} + [\cos t + \sin t)\vec{j}] \cdot [-\sin t\, i + \cos t\, j]\, dt$$

$$= 4 \int_0 (-\sin t \cos t + \sin^2 t + \cos^2 t + \sin t \cos t)\, dt$$

$$= 4 \int_0^{3\pi/2} dt = 4 \cdot \frac{3\pi}{2} = \boxed{6\pi}$$

3.) Compute the line integral of $\vec{H} = (6x + y^2)\vec{i} + (2xy)\vec{j}$ along a counterclockwise path around a circle of radius 2, centered at the origin, starting at (2, 0) and ending at (0, -2).

$$\text{Curl } \vec{H} = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2y - 2y = 0 \qquad \text{so } \vec{F} = \vec{\nabla} f$$

$$\frac{\partial f}{\partial x} = 6x + y^2 \qquad f = 3x^2 + xy^2 + C(y)$$

$$\frac{\partial f'}{\partial y} = 2xy + c'(y) = 2xy \qquad c'(y) = 0 \qquad c_y = c$$

$$f = 3x^2 + xy^2$$

$$\int \vec{F} \cdot d\vec{r} = f(Q) - f(P) = 3x^2 + xy^2 \Big|_{2,0}^{0,-2}$$

$$= [3(0)^2 + (0)(-2)^2] - [3(2)^2 + 2(0)^2] = \boxed{-12}$$