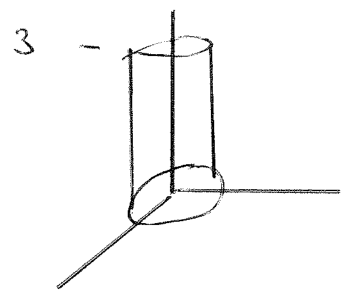
MA3160
Quiz 11 – Summer 2007
14th August 2007
Due: 16th August 2007
Open Book/Notes

Name: Key

1.) Calculate the flux of the vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ through a <u>closed</u> cylindrical surface of radius 2 and height 3 centered on the z-axis with its base in the xy plane by the following methods.

a.) Using a surface integral after the methods in Chapter 19.



2 disks + cylindrical surface

top disk @ $z = 3$ $\quad \vec{F} = x\vec{i} + y\vec{j} + 3\vec{k}$

bot disk @ $z = 0$ $\quad \vec{F} = x\vec{i} + y\vec{j}$

Flux thru bottom disk $= \int \vec{F} \cdot d\vec{A} = \int (x\vec{i} + y\vec{j}) \cdot (-\vec{k}\, dA) = 0$

Flux " top disk $= \int (x\vec{i} + y\vec{j} + 3\vec{k}) \cdot (\vec{k}\, dA) = \int 3\, dA = 3A = 12\pi$

$A = \pi r^2 = 4\pi$

Flux thru cylinder $= \int [2\cos\theta\,\vec{i} + 2\sin\theta\,\vec{j} + z\vec{k}] \cdot [\cos\theta\,\vec{i} + \sin\theta\,\vec{j}]\, 2\, dz\, d\theta$

$= 2 \int (\sin^2\theta + \cos^2\theta)\, 2\, dz\, d\theta = 4 \int_0^{2\pi} \int_0^3 dz\, d\theta$

$= \cdot 2\pi \cdot 3 =$

Total Flux = Flux thru walls + Flux thru ends $: 0 + 12\pi + 24\pi = \boxed{36\pi}$

b.) Using the Divergence Theorem.

$\operatorname{div} \vec{F} = 1 + 1 + 1 = 3$

$\int \vec{F} \cdot d\vec{A} = \int \operatorname{div} \vec{F}\, dV$

$= \int 3\, dV = 3V = 3\pi r^2 h = \boxed{36\pi}$

2.) Use the Divergence Theorem to calculate the flux of the vector field $\vec{F} = x^2\vec{i} + y^2\vec{j} + e^{xy}\vec{k}$ through a rectangular solid of length = 2, width = 3, and height = 5 that is located in the first octant with one corner at the origin and sides parallel to the coordinate axes.

$$\text{div } \vec{F} = 2x + 2y$$

$$\int \vec{F} \cdot d\vec{A} = \int \text{div } F \, dV$$

$$= \int_0^5 \int_0^2 \int_0^3 (2x+2y)\, dx\, dy\, dz$$

$$= \int_0^5 \int_0^2 \left(x^2 + 2xy \Big|_0^3 \right) dy\, dz$$

$$= \int_0^5 \int_0^2 (9 + 6y)\, dy\, dz$$

$$= \int_0^5 (9y + 3y^2)\Big|_0^2 \, dz$$

$$= \int_0^5 30\, dz = \boxed{150}$$