1. Match the name of the surface with a plausible equation.

   a. (2 pts) $z = y^2$, $x$ free to vary               i. circular cylinder
   b. (2 pts) $x^2 + \frac{y^2}{4} + z^2 = 9$            ii. hyperbolic paraboloid
   c. (2 pts) $x^2 + y^2 = 4$, $z$ free to vary         iii. paraboloid
   d. (2 pts) $z = -\sqrt{x^2 + y^2}$                   iv. single cone
   v. parabolic cylinder                                 v. sphere
   vi. sphere                                            vii. ellipsoid

2. (5 pts) Write a linear function with the given points.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td>200</td>
<td>2</td>
</tr>
<tr>
<td>300</td>
<td>1</td>
</tr>
<tr>
<td>400</td>
<td>0</td>
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</table>

<table>
<thead>
<tr>
<th>x</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
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</tr>
<tr>
<td>400</td>
<td>6</td>
<td>9</td>
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</tbody>
</table>

3. (5 pts) (Sketch the level surface of the function $g(x, y, z) = x^2 + y^2 + z$ when $g$ is set equal to the constant 1.)

(OVER)
4. Given the plane \( z - 5(x-2) = 3(5-y) \).

\[ \hat{n} = \] a. (3 pts) Find the normal vector to the plane.

b. (4 pts) Graph the plane on the given axes and indicate the values of the x-, y-, and z-intercepts.

5. Let \( \hat{v} = 2\hat{i} - 3\hat{j} - \hat{k} \) and \( \hat{w} = 5\hat{i} + 3\hat{j} - \hat{k} \).

\[ \] a. (3 pts) Find \( \hat{v} \cdot \hat{w} \).

\[ \] b. (2 pts) Are the two vectors parallel, perpendicular or neither?

c. (3 pts) Find \( \hat{v} \times \hat{w} \). Calculators can be used only for checking.

d. (2 pts) Find \( \tan\theta \) where \( \theta \) is the angle between \( \hat{v} \) and \( \hat{w} \).
6. Find the following partial derivatives.

a. (4 pts) \( z_x \) if \( z = (3xy + 2x)^5 \)

b. (4 pts) \( \frac{\partial f}{\partial y} \) if \( f(x,y) = e^{2xy} \)

c. (4 pts) \( \frac{\partial f}{\partial x} \bigg|_{\left(\frac{\pi}{3},1\right)} \) if \( f(x, y) = \frac{\sin xy}{y} \)

d. (4 pts) \( g_{xy} \) if \( g(x,y) = 3x^2 + 2x \ln y \)

7. Let \( f(x,y) = x^2 + y^3 \)

a. (4 pts) Find the local linearization of \( f(x,y) \) at the point \((1,2)\).

b. (2 pts) Estimate \( f(1.04, 1.98) \) using the linearization.
8. Let \( f(x,y) = x^2 + \ln y \) and let point \( P = (3,1) \).

a. (4 pts) Find \( \nabla f \) at the point \( P \).

b. (2 pts) In what direction from the point \( P \) is \( f \) increasing the fastest?

c. (4 pts) Find the directional derivative of \( f \) at the point \( P \) in the direction of \( \hat{u} = \frac{1}{\sqrt{2}} \hat{i} - \frac{1}{\sqrt{2}} \hat{j} \).

9. Let \( w = f(x,y,z) = x^2 + 2y + z \) where \( x = u^2 + v^2 \), \( y = u^2 v^2 \) and \( z = u \).

a. (3 pts) Draw a tree diagram to the right expressing the relationships between the variables.

b. (4 pts) Find \( \frac{\partial f}{\partial v} \).
10. Use the level curve of the function $z = f(x, y)$ to decide the sign (+, -, 0) of each of the following partial derivatives.

__________ a. (2 pts) $f_x$

__________ b. (2 pts) $f_{xx}$

__________ c. (2 pts) $f_y$

__________ d. (2 pts) $f_{yy}$

11. (6 pts) Find the critical points of the function $f(x, y) = x^3 + y^2 - 3xy$ and classify each as a local max, a local min, a saddle point or none of these.

<table>
<thead>
<tr>
<th>Critical Points</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y) = ( , )$</td>
<td></td>
</tr>
<tr>
<td>$(x, y) = ( , )$</td>
<td></td>
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</tbody>
</table>

(OVER)
12. Given the contour diagram to the right. Identify each labeled point as a local minimum, a local maximum, a saddle point or none of these.

a. (2 pts) point A
b. (2 pts) point B
c. (2 pts) point C

13. (6 pts) Use the method of Lagrange multipliers to find the maximum value of the function

\[ f(x, y) = -3x^2 - 2y^2 \]

subject to the constraint that \( x \) and \( y \) lie on the parabola

\[ y - x^2 = -2. \]

max value of \( f = \) \[ \] and occurs at \( (x, y) = (\) \[ \), \[ ) \]