1.) Find line integral \( \int_C \vec{F} \cdot d\vec{r} \) given \( \vec{F} = -y \hat{i} + x^2 \hat{j} \) and \( C \) is open path, \( y = \sin(x) \), between the points \((0, 0)\) and \((\pi, 0)\). (12 pts)

2.) Compute the flux of the vector field \( \vec{v} = \sqrt{x^2 + y^2} \hat{k} \) through a disk with a radius of 2 meters, centered on the z-axis, in the plane \( z = 3 \), oriented upward. Include units in your answer given that \( \vec{v} \) is in m/s. (12 pts)
3.) Compute the average temperature of the region that is bounded by the cone $z = r$ and the plane $z = 4$. The temperature distribution inside of the cone is $T = r$. Include proper units given the temperature is in units of Deg C and the length dimensions are in meters. Note that the volume of a cone is $V = \frac{\pi r^2 h}{3}$. Use cylindrical coordinates. (12 pts)

4.) Use Green’s Theorem to calculate the line integral around a triangular path connecting the points (0,0), (1,0), (1,2) and (0,0), in that order, for the field, $\vec{F} = e^y \vec{i} + xy \vec{j}$. (12 pts)
5.) For the function \( f(x,y) = 25 - (x^2 - 2x + y^2 + 4y) \), address the following questions.

a.) Find the local linearization at the point (-1,2). \( \text{(5 pts)} \)

b.) Find and classify any critical points. \( \text{(5 pts)} \)

c.) Sketch a contour diagram for \( f(x,y) \) on the figure below. \( \text{(2 pts)} \)
6.) The following questions pertain to the vector field plot shown below. (2 pts each)

a.) Circle the number of the equation below that is most consistent with the appearance of the field.
   
   \[ \vec{F} = x\hat{i} + y\hat{j} \]  
   \[ \vec{F} = x\hat{i} + y^2\hat{j} \]  
   \[ \vec{F} = 2\hat{i} + y^2\hat{j} \]  
   \[ \vec{F} = x\hat{i} - y^2\hat{j} \]

b.) Show, algebraically (analytically) that the field above is path-independent.

c.) Explain how the result calculated in Part (b.) could have been anticipated from the field plot (i.e., graphically).

d.) Find the potential function that corresponds to the field shown in the plot.

e.) Use the result above to find the line integral along a spiral path starting at (0, -1) and ending at (2,0).

f.) Does is the sign of your answer consistent with what would be expected? Explain.
7.) Use the Divergence Theorem to compute the flux integral \( \iiint_S \mathbf{F} \cdot d\mathbf{A} \) through a sphere of radius 2, centered at the origin, and \( \mathbf{F} = x \mathbf{i} + 2y \mathbf{j} + e^{\sin(x)} \mathbf{k} \). (14 pts)

8.) Use Stokes' Theorem to evaluate \( \oint_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is the perimeter of a rectangle formed by connecting the points (1,1,0), (0,1,2), (0,-1,2) and (1,-1,0), in that order, where \( \mathbf{F} = z \mathbf{i} + x \mathbf{j} + y^2 \mathbf{k} \). Note that the rectangle lies in the plane \( 4x + 2z = 4 \). (14 pts)
Formula Sheet for Final Exam

**Classification of Critical Points**

\[ D(x,y) = f_{xx}f_{yy} - (f_{xy})^2 \]

**Criteria**

- \( D > 0 \) and \( f_{xx} > 0 \) → local minimum
- \( D > 0 \) and \( f_{xx} < 0 \) → local maximum
- \( D < 0 \) → saddle point
- \( D = 0 \) → undetermined

**Flux through Surfaces**

\[ f(x,y): \quad \int_S \vec{F} \cdot d\vec{A} = \int_R \vec{F}(x,y,f(x,y)) \cdot (-f_x \hat{i} - f_y \hat{j} + \hat{k}) dxdy \]

**Cylindrical Surface:**

\[ \int_S \vec{F} \cdot d\vec{A} = \int_R \vec{F}(R,\theta,z) \cdot (\cos \theta \hat{i} + \sin \theta \hat{j}) Rd\theta \]

\[ x = R \cos \theta \quad y = R \sin \theta \quad z = z \]

**Sphere:**

\[ \int_S \vec{F} \cdot d\vec{A} = \int_R \vec{F}(R,\theta,\phi) \cdot (\sin \phi \cos \theta \hat{i} + \sin \phi \sin \theta \hat{j} + \cos \phi \hat{k}) R^2 \sin \phi d\phi d\theta \]

\[ x = R \sin \phi \cos \theta \quad y = R \sin \phi \sin \theta \quad z = R \cos \phi \]

**Integration Formulas**

\[ \int x \sin(ax) \, dx = \frac{\sin(ax)}{a^2} - \frac{x \cos(ax)}{a} \]

\[ \int x \cos(ax) \, dx = \frac{\cos(ax)}{a^2} + \frac{x \sin(ax)}{a} \]

\[ \int x^2 \sin(ax) \, dx = \frac{2x}{a^2} \sin(ax) + \left( \frac{2}{a^3} - \frac{x^2}{a} \right) \cos(ax) \]

\[ \int x^2 \cos(ax) \, dx = \frac{2x}{a^2} \cos(ax) + \left( \frac{x^2}{a} - \frac{2}{a^3} \right) \sin(ax) \]