Chapter 12

Section 12.1

2. A closest to yz plane.
   B on xz plane
   C farthest away from xy

4. (1, -1, 1)

6. \( \sqrt{2} \)

8. \((x-1)^2 + (y-2)^2 + (z-3)^2 = 25\)

10. Plane parallel to xz plane and passing thru \((0, 1, 0)\)

22. (a.) \[ C = f(d, m) = 40d + 0.15m \]
    (b.) \[ f(5, 3w) = 40(5) + 0.15(3w) = 295 \]

Section 12.2

2. (a) I  (b) V  (c) IV  (d) III  (e) III

4. Sphere with \( r = 3 \), centered at origin

6. Paraboloid opening \( \downarrow \) with vertex at \( z = 5 \)

8. (unlabeled)
Chapter 12

Section 12.3

(4.)

Circles become more closely packed as \( c \downarrow \)

120  
(a) \( \neq 137 \)  
(b) \( \neq 6250 \)  
(c)  
3 5 7 9 11 13 15  
7.5 7.7 6.9 6.5 6.25 6.5 7.75

Section 12.4

4) Linear  \( \frac{\Delta y}{\Delta z} = 3 \)  \( \frac{\Delta x}{\Delta z} = -4 \)

6) Linear  \( \frac{\Delta y}{\Delta z} = 5 \)  \( \frac{\Delta x}{\Delta z} = 2 \)

12) Non linear  
14) Linear

Section 12.5

16) \(-x^2 + y^2 - z^2 = 0 \)  \( \Rightarrow \) cone

18) \(x^2 + y^2 = 1 \)  \( \Rightarrow \) cylindrical surface
Chapter 13

Section 13.3

2. \( -38 \)

3. \( 238 \)

Section 13.4

2. \( \mathbf{j} - \mathbf{k} \)

3. \( \mathbf{i} - \mathbf{j} \)

10. \( -2\mathbf{\hat{h}} \)

12. \( -2\mathbf{i} - 7\mathbf{j} - 13\mathbf{\hat{h}} \)

14. \( -x + y + z = 3 \)

15. \( \tan \theta = \frac{\sqrt{38}}{3} \)
Section 14.1

4. (a) $f_p < 0$

(b) $f_a(8, 12) = 150$

Change in unit sales with ad spending
up by 150 when price is 8 and spending is 12.

Section 14.2

6. $\frac{2V}{\delta r} = \frac{2}{3} \pi r h$

8. $\frac{\partial}{\partial T} \left( \frac{2\pi r}{T} \right) = -\frac{2\pi r}{T^2}$

11. $\frac{1}{2} v^2$

18. $\frac{2\pi}{V}$

26. $(15x^2 - 3y^3) - \cos(5x^3y - 3xy^2)$

34. 13.6
Section 14.3

8 \[ dh = e^{-3t} \cos(x + 5t) \, dx + (5e^{-3t} \cos(x + 5t) - 3\sin(x + 5t) \cdot e^{-3t}) \, dt \]

10 \[ dF = 0.01 \, G \, dm - 0.2 \, G \, dr \]

14 \[ z = 12x - 6y - 7 \]

16 \[ df = \frac{1}{3} \, dx + 2 \, dy \]

\[ df \approx 2.973 \]

Section 14.4

30 \[ \nabla f(5, 2) = 50 \hat{i} + 96 \hat{j} \]

52 \[ (a) \quad \frac{2}{\sqrt{13}} \quad (b) \quad \frac{1}{\sqrt{17}} \]

Section 14.5

1 \[ \nabla f = -\sin(x + y) \hat{i} + (-\sin(x + y) + \cos(y + z)) \hat{j} + \cos(y + z) \hat{k} \]
Chapter 14

Section 14.6

2. \( \frac{dz}{dt} = \frac{t^3 - 2}{t^4 + t} \)

16. \( \frac{dw}{dt} = \frac{dw}{dx} \frac{dx}{dt} + \frac{dw}{dy} \frac{dy}{dt} + \frac{dw}{dz} \frac{dz}{dt} \)

20. \( \frac{dp}{dt} = -5 \text{ Pa/hr} \)

Section 14.7

4. \( f_x = e^y \quad \quad f_y = xe^y \)
   \( f_{xx} = 0 \quad \quad f_{yy} = xe^y \)
   \( f_{xy} = e^y \quad \quad f_{yx} = e^y \)

6. \( f_x = e^y \quad \quad f_y = xe^y + ye^y + e^y \)
   \( f_{xx} = 0 \quad \quad f_{yy} = xe^y + ye^y + 2e^y \)
   \( f_{xy} = e^y \quad \quad f_{yx} = e^y \)

10. \( f_x = 6 \cos 2x \cos 5y \quad \quad f_y = -15 \sin 2x \sin 5y \)
    \( f_{xx} = -12 \sin 2x \cos 5y \quad \quad f_{yy} = -75 \sin 2x \cos 5y \)
    \( f_{xy} = -30 \cos 2x \sin 5y \quad \quad f_{yx} = -30 \cos 2x \sin 5y \)

14. \( Q(x,y) = 1 - 2x + y + 4x^2 - 4xy + y^2 \)

16. \( Q(x,y) = 1 - \frac{1}{2} x^2 - 3xy - 9y^2 \)

13. \( Q(x,y) = 1 + 2x - \frac{1}{2} y^2 \)
Section 14.7

26. \( f_x < 0 \)
\( f_y < 0 \)
\( f_{xx} = f_{yy} = f_{xy} = 0 \)

30. \( f(x, y) = (x + 2y)^y \)
Chapter 13

Section 13.1

(2) \( \text{CP at } (2,2) \)
\[ D = 8 \]
\[ f_{xx} > 0 \]
local min

Section 13.3

(5) CP's at \((6,-2)\) => local max \((-6,2)\) => local min

(10) CP at \((1,1,1,2)\) => max

(14) CP's \((0,1), (0,-1), (1,0), (-1,0), \left(-\frac{2}{3}, \frac{11}{3}\right)\)

max/min values are 1, -1.

Inside circle CP = \((0,0)\)
and is a saddle pt.

(12) CP at \(q_1 = 50\) \(q_2 = 150\)
and is a min value.

22 (a) 25, 219
(b) \(z = 11.348\)
Chapter 16

Section 16.1

(a) \( A_{xy} = 3702 \)
(b) 77,125

10. function is always \((+)_x\), so \(S\) is \((+)_x\).

20. \( \cos y \ (+) \) on interval so \(S\) is \((+)\)

Section 16.2

4. \( \frac{2}{3} \)

6. \( \int_0^1 \int_{3x}^{2x} f(x, y) \, dy \, dx \) or \( \int_0^1 \int_{3y}^{y} f(x, y) \, dx \, dy \)

10. \( \int_0^1 \int_{\frac{x-1}{3}}^y f(x, y) \, dy \, dx \)

14. 656.1

12. 2.38

23. 0.23

36. \( \int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} (9-x^2-y^2) \, dx \, dy \)

38. 4
Section 16.3

30 integal is positive

32 \[ S_{w_y} \, dv = 0 \]

34 \[ S_{w_x} \, dv = 0 \]

36 \[ S (2-2) \, dv < 0 \]

Section 16.4

4 \[ \int_{\pi/4}^{3\pi/4} \int_{0}^{2} f(r, \theta) \, dr \, d\theta \]

2

\[ \frac{38 \pi}{9} \]

14 \[ 3 \left( \frac{2\pi}{3} \right) \]

20 \[ \frac{3 \left( 2\pi \left( \sqrt{2} - 1 \right) \right)}{9} \]

Note break problem into 2 parts

Total = \( \nabla \) + \( \Delta \)
Chapter 16

Section 16.5

2. \( V = \frac{200\pi}{3} \)

4. \( \int_0^5 \int_0^{2\pi} \int_{\pi/2}^{\pi} \rho \sin \phi \; d\phi \; d\theta \; d\rho = 25\pi \)

28. Cylindrical
   \( V = \int_0^{2\pi} \int_0^1 \int_0^{\sqrt{1-z^2}} r \; dz \; dr \; d\theta \)

Spherical
   \( V = \int_0^{\pi/2} \int_0^{2\pi} \int_0^{\rho \sin \phi} \rho^2 \sin \phi \; d\rho \; d\phi \; d\theta \)
Section 17.1

1) Lines don't intersect

2) \( t_1 = -2 \), \( t_2 = 1 \)
   \( x = -7 \), \( y = 7 \), \( z = -4 \)

3b) upper arc \( 5\hat{i} + 5(-\cos t\hat{i} + \sin t\hat{j}) \)
    lower arc \( 5\hat{i} + 5(\cos t\hat{i} + \sin t\hat{j}) \)

5a) helix

Section 17.2

2) \( \vec{V} = 3\hat{i} + \hat{j} - \hat{k} \)
   \( \vec{a} = x''\hat{i} + y''\hat{j} + z''\hat{k} = \vec{0} \)

3) \( \vec{V} = 6 \sin t \cos t \hat{i} - \sin t \hat{j} + 2t \hat{k} \)
   \( \| \vec{V} \| = \sqrt{36 \sin^2 t \cos^2 t + \sin^2 t + 4t^2} \)

particle at rest when \( t = 0 \)
Chapter 17

Section 17.2

(2) \[ \vec{V} = -6\pi \sin((2\pi t) \hat{i}) + 6\pi \cos((2\pi t) \hat{j}) \]
\[ \hat{a} = 12\pi^2 \cos((2\pi t) \hat{i}) - 12\pi^2 \sin((2\pi t) \hat{j}) \]
\[ \vec{V} \cdot \vec{a} = 0 \]
\[ ||\vec{V}|| = 6\pi \]
\[ ||\vec{a}|| = 12\pi^2 \]

(14) \[ \vec{V} = 2t (\hat{i} - 2\hat{j} - \hat{k}) \]
\[ \vec{a} = 2 (\hat{i} - 2\hat{j} - \hat{k}) \]
\[ ||\vec{V}|| = 2\sqrt{5} |t| \]

(30) a) vertical velocity = 2
b) \[ \vec{V}_z = 2t = 10 \quad \boxed{t = 5} \]

c) \[ \vec{V}(5) = -\sin(5) \hat{i} + \cos(5) \hat{j} + 2 \hat{k} \]

Remember to use radians.

d) \[ r(t) = 0.284\hat{i} - 0.959\hat{j} + 10\hat{k} + (t-5) (0.959\hat{i} + 0.284\hat{j} + 2\hat{k}) \]

Section 17.3

(2) \[ \vec{F} = C\hat{y} \hat{c} \quad c < 1 \]

(4) \[ \vec{F} = -C_1\hat{x} + C_2\hat{y} \hat{c} \quad C_1, C_2 > 0 \]

\[ ||\vec{F}|| \quad \boxed{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow} \]
\[ \boxed{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow} \]

(10) (a) IV (b) III (c) I (d) II
Chapter 18

Section 18.1

2. Negative

0 Zero

6 $\int F \cdot dr = 0$

8 28

15 15

32 32

Section 18.2

4 $\frac{82}{3}$

111,664

8 $-2\pi$

14 21

\[
\begin{align*}
\int_{C_1} &= 0 \\
\int_{C_2} &= -\frac{2}{6} \\
\int_{C_3} &= \eta \\
\text{total} &= 1
\end{align*}
\]
Chapter 18

Section 18.3

2. a) 1  b) i  c) 1

4. Vector field is gradient field

8. path independent

10. path dependent

12. -7  10  0

28. -0.3

Section 18.4

4. \( f = \frac{x^3}{3} + xy^2 + C \)

6. \( f = \ln k (xyz) \) when \( k > 0 \)

12. \( \frac{\pi}{2} \)

22. \( \frac{-3\pi}{2} \)
Chapter 19

Section 19-1

10.  $\text{Flux} = 0$

22.  $\frac{8L\pi}{2}$
Chapter 20

Section 20.2

2) \[ \text{Flux} = 3 \cdot 2 \cdot 2 \cdot 2 = 24 \]

4) \[ 36\pi \]

Section 20.3

4) Flow diverging but not rotating

14) \[ \vec{\mathbf{F}} = z \hat{\mathbf{j}} + x \hat{\mathbf{k}} \]

a) at \((0, -1, 0)\) \[ \text{and } \vec{\mathbf{F}} = 0 \]

b) Field rotational since \[ \text{curl } \vec{\mathbf{F}} \neq 0 \] everywhere.

Section 20.4

23) a) \[ \text{Flux} = -2\pi a^2 \]

b) \[ \text{Flux} = 2\pi a^2 \]

c) Difference is due to circulation specified in opposite directions.