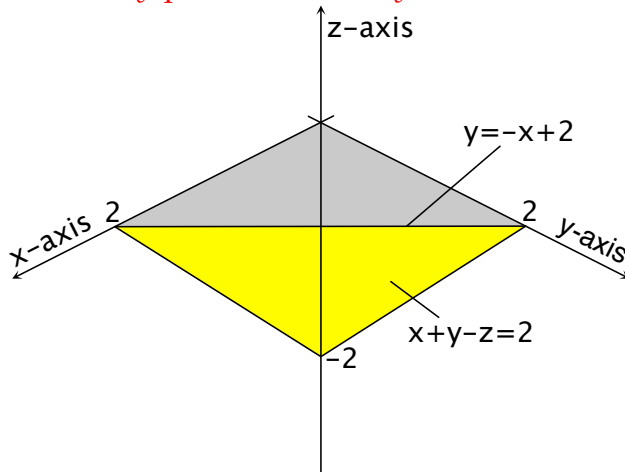


1. (5pts.) A solid region is bounded by the xy -plane, the xz -plane, the yz -plane, and the plane $x + y - z = 2$. Write an iterated integral (do **not** evaluate) which gives the mass of this solid if the density is $\delta(x, y, z)$.

Solution

Note that the z -values are negative in this region. The intersection of the plane $x + y - z = 2$ and the xy -plane is the line $y = -x + 2$.



The mass is given by

$$M = \int_W \delta(x, y, z) dV,$$

which leads to

$$M = \int_{x=0}^2 \int_{y=0}^{-x+2} \int_{z=x+y-2}^0 \delta(x, y, z) dz dy dx$$

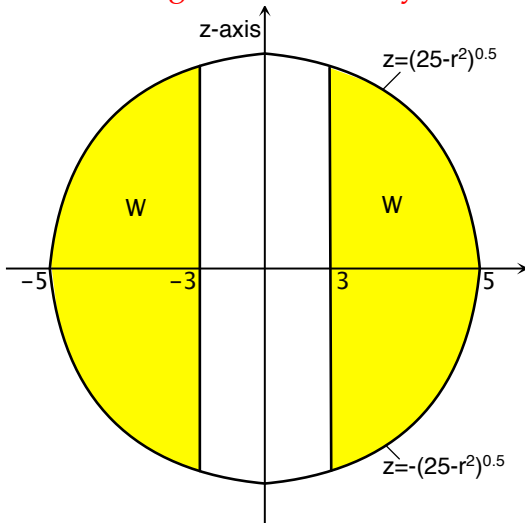
Note the order of integration!

2. (5pts.) Use cylindrical coordinates to compute the volume of a sphere of radius 5 mm with a hole of radius 3 mm through its center. (Hint: let the axis of the hole coincide with the z -axis.)

Solution

$$V = \int_W dV = \int_W r dz dr d\theta = \int_{\theta=0}^{2\pi} \int_{r=3}^5 \int_{z=-\sqrt{25-r^2}}^{\sqrt{25-r^2}} r dz dr d\theta = \int_{\theta=0}^{2\pi} \int_{r=3}^5 2r \sqrt{25-r^2} dr d\theta$$

The dr integral is solved by substituting $w = 25 - r^2$ and $dw = -2r dr$ which leads to



$$-2\pi \int_{r=3}^5 \sqrt{w} dw = -2\pi \frac{2}{3} (25 - r^2)^{3/2} \Big|_{r=3}^5$$

Therefore,

$$V = \int_{\theta=0}^{2\pi} \int_{r=3}^5 \int_{z=-\sqrt{25-r^2}}^{\sqrt{25-r^2}} r dz dr d\theta = 256\pi/3$$