Date: Oct 26, 2009

MA3160-04 Quiz 6

No Calculators! Justify all answers! **Name**(print): **Solutions**

1. (5pts.) Evaluate $\int_{y=0}^{2}\int_{x=y/2}^{1} e^{x^2}\,dx\,dy$ by reversing the order of integration.

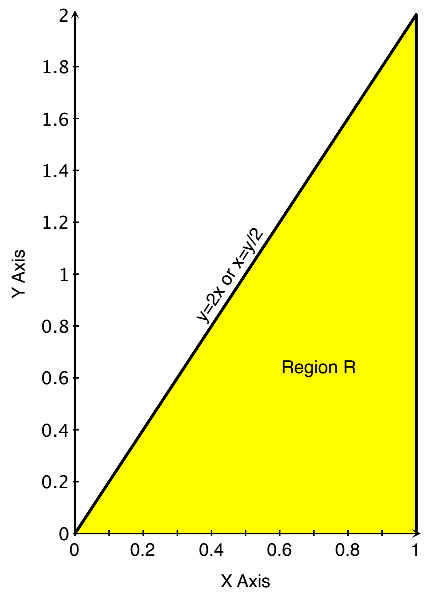
Solution

$$\int_{y=0}^{2}\int_{x=y/2}^{1} e^{x^2}dxdy = \int_{x=0}^{1}\int_{y=0}^{2x} e^{x^2}dydx =$$

$$\int_{x=0}^{1}\left[ye^{x^2}\right]_{y=0}^{2x}dx = \int_{x=0}^{1} 2xe^{x^2}dx = \left[e^{x^2}\right]_{x=0}^{1} = e-1$$

The last integral is solved by substituting $w = x^2$ and $dw = 2xdx$. Therefore,

$$\boxed{\int_{y=0}^{2}\int_{x=y/2}^{1} e^{x^2}dxdy = e-1}$$



2. (5pts.) Find the volume of the region under the graph of $x + y + z = 1$ in the first octant.

Solution

The first octant is the region $\{(x,y,z)|x \geq 0, y \geq 0, z \geq 0\}$. The intersection of the surface (plane) $x + y + z = 1$ with the xy-plane is the line $y = 1 - x$. Therefore,

$$\text{Volume} = \int_0^1\int_{y=0}^{y=1-x}(1-x-y)dydx = \int_0^1\left[y - xy - \frac{y^2}{2}\right]_{y=0}^{y=1-x}dx$$

$$= \int_0^1\left[(1-x) - x(1-x) - \frac{(1-x)^2}{2}\right]_{y=0}^{y=1-x}dx = \int_0^1\left(\frac{1}{2} - x + \frac{x^2}{2}\right)dx = \left[\frac{x}{2} - \frac{x^2}{2} + \frac{x^3}{6}\right]_0^1 = \frac{1}{6}$$

$$\boxed{\textbf{Volume} = \frac{1}{6}}$$