Date: Oct 07, 2009

MA3160-04 Quiz 5

No Calculators! Justify all answers! **Name**(print): **Solutions**

---

1. (4pts.) The air pressure is dropping at a constant rate with respesctc to time everywhere. In the eastward direction, the air pressure decreases at a rate of 1 pascal per kilometer. A ship sailing eastward at 15 km/hr past an island records a pressure drop of 40 pascals in 2 hours. Estimate the time rate of change of the air pressure on the island.

Solution

Let $p(x,t)$ be the air pressure at location x and time t. We want the time rate of change of the air pressure on the island (at fixed x) which is $\frac{\partial p(x,t)}{\partial t}$. The pressure drop measured on the boat is the time change for varying x and varying t, that is, $\frac{\mathrm{d}p(x,t)}{\mathrm{d}t} = -40/2 = -20$. Further, the speed of the boat is $\frac{\partial x}{\partial t} = 15$, and the pressure drop in the eastward direction (at constant t) is $\frac{\partial p(x,t)}{\partial x}$. Therefore, by the chain rule,

$$\frac{\mathrm{d}p(x,t)}{\mathrm{d}t} = \frac{\partial p}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial p}{\partial t} \quad \text{which leads to} \quad \boxed{\frac{\partial p}{\partial t} = -20 - (-1 \cdot 15) = -5\ Pa/hr.}$$

2. (6pts.) Let $f(x,y) = \sqrt{2x+y}$. Find the quadratic Taylor polynomial $Q(x,y)$ valid near (0,1).

$$Q(x,y) = f(0,1) + f_x(0,1)x + f_y(0,1)(y-1) + \frac{1}{2}f_{xx}(0,1)x^2 + f_{xy}(0,1)x(y-1) + \frac{1}{2}f_{yy}(0,1)(y-1)^2$$

$$f(0,1) = 1$$

$$f_x(0,1) = \left.\frac{2}{2\sqrt{2x+y}}\right|_{(0,1)} = 1$$

$$f_y(0,1) = \left.\frac{1}{2\sqrt{2x+y}}\right|_{(0,1)} = \frac{1}{2}$$

$$f_{xx}(0,1) = \left.\frac{-2}{2(2x+y)^{3/2}}\right|_{(0,1)} = -1$$

$$f_{xy}(0,1) = \left.\frac{-1}{2(2x+y)^{3/2}}\right|_{(0,1)} = \frac{-1}{2}$$

$$f_{yy}(0,1) = \left.\frac{-1}{4(2x+y)^{3/2}}\right|_{(0,1)} = \frac{-1}{4}$$

Therefore, the quadratic Taylor polynomial is

$$\boxed{Q(x,y) = 1 + x + \frac{1}{2}(y-1) - \frac{1}{2}x^2 - \frac{1}{2}x(y-1) - \frac{1}{8}(y-1)^2}$$