Date: Oct 02, 2009

MA3160-04 Quiz 4

No Calculators! Justify all answers! **Name**(print): **Solutions**

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1. **(5pts.)** Find the directional derivative of $f(x,y,z) = xz - y^2$ at the point $P = (1,1,0)$ in the direction of the point $Q = (1,0,1)$.

The direction vector is

$$\vec{v} = \vec{PQ} = (1-1)\vec{i} + (0-1)\vec{j} + (1-0)\vec{k} = -\vec{j} + \vec{k}$$

with length

$$\|\vec{v}\| = \sqrt{0^2 + (-1)^2 + 1^2} = \sqrt{2}.$$

The *unit* direction vector is

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{-1}{\sqrt{2}}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}.$$

The gradient of $f(x,y,z)$ at $P = (1,1,0)$ is

$$\textbf{grad}\, \boldsymbol{f}(\mathbf{1},\mathbf{1},\mathbf{0}) = f_x(1,1,0)\vec{i} + f_y(1,1,0)\vec{j} + f_z(1,1,0)\vec{k} = (z\vec{i} - 2y\vec{j} + x\vec{k})|_{(1,1,0)} = -\mathbf{2}\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}.$$

Therefore, the derivative in the direction $\vec{u}$ at the point $P = (1,1,0)$ is

$$\boxed{f_{\vec{u}}(1,1,0) = \textbf{grad}\, f(1,1,0)\cdot\vec{u} = \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}}$$

2. **(5pts.)** Find the equation of the tangent plane at $P = (1,2,3)$ to the surface $x^2 + y^2 - xyz = -1$

Note that the point $P = (1,2,3)$ actually lies on the surface $x^2 + y^2 - xyz = -1$. Why?
The surface $x^2 + y^2 - xyz = -1$ is a level surface of the function

$$w = f(x,y,z) = x^2 + y^2 - xyz.$$

Therefore, the normal vector at $P = (1,2,3)$ to the surface $x^2 + y^2 - xyz = -1$ is

$$\text{grad}\, f(1,2,3) = f_x(1,2,3)\vec{i} + f_y(1,2,3)\vec{j} + f_z(1,2,3)\vec{k} = [(2x-yz)\vec{i} + (2y-xz)\vec{j} - xy\vec{k}]|_{(1,2,3)} = -4\vec{i} + \vec{j} - 2\vec{k}.$$

Let $Q = (x,y,z)$ be a general point on the tangent plane. Then the vector

$$\vec{v} = \vec{PQ} = (x-1)\vec{i} + (y-2)\vec{j} + (z-3)\vec{k}$$

lies on the tangent plane and is perpendicular to grad $f(1,2,3)$. Therefore, grad $f(1,2,3)\cdot\vec{v} = 0$ gives the equation of the tangent plane, i.e.,

$$\boxed{\mathbf{-4(x-1) + (y-2) - 2(z-3) = 0}}$$