

Date: Sep 25, 2009

## MA3160-04 Quiz 3

No Calculators! Justify all answers!

Name(print):

Solutions

Let  $f(x, y) = \sqrt{x^3 + y^2}$ .

(a) (3pts.) Use a difference quotient to approximate  $f_y(0, 1)$  with  $h = 0.01$ .

The difference quotient corresponding to  $f_y(0, 1)$  is

$$\frac{f(0, 1+h) - f(0, 1)}{h} = \frac{\sqrt{0^3 + 1.01^2} - \sqrt{0^3 + 1^2}}{0.01} = \frac{1.01 - 1}{0.01} = 1$$

(b) (4pts.) Find the differential of  $f(x, y)$  at the point (2,1).

Note that

$$f_x(x, y) = \frac{3x^2}{2\sqrt{x^3 + y^2}} \quad \text{and} \quad f_y(x, y) = \frac{y}{\sqrt{x^3 + y^2}}$$

Therefore,

$$df = f_x(x, y)dx + f_y(x, y)dy = \frac{3x^2}{2\sqrt{x^3 + y^2}}dx + \frac{y}{\sqrt{x^3 + y^2}}dy$$

and at the point (2,1)

$$df = 2dx + \frac{1}{3}dy$$

(c) (3pts.) Use part (b) to estimate  $f(2.1, 0.9)$ .

$$f(2.1, 0.9) \approx f(2, 1) + f_x(2, 1)\Delta x + f_y(2, 1)\Delta y = 3 + 2(0.1) + \frac{1}{3}(-0.1)$$

Therefore,

$$f(2.1, 0.9) \approx 3\frac{1}{6} \approx 3.1667$$