

Date: Sep 09, 2009

# MA3160-04 Quiz 1

No Calculators! Justify all answers!

Name(print):

Solutions

---

1. Consider the sphere of radius 3 centered at  $P=(-1, 2, -1)$ .

(a) (2pts.) Give the equation of this sphere.

$$(x + 1)^2 + (y - 2)^2 + (z + 1)^2 = 9$$

(b) (2pts.) Find the equations of the planes which are parallel to the  $xy$ -plane and touch the sphere.

A plane parallel to the  $xy$ -plane has equation  $z = \text{const.}$ , where the constant is a  $y$ -coordinate of the point of contact. There are two planes of contact with the equations

$$z = 2 \quad \text{and} \quad z = -4$$

(c) (2pts.) Give the coordinates of the points of contact of the sphere and the planes in part (b).

There are two points of contact with  $z$ -coordinates  $z = 2$  and  $z = -4$ . The  $x$ -coordinate and the  $y$ -coordinate are the ones from the center of the sphere. Therefore,

$$P_1 = (-1, 2, 2) \quad \text{and} \quad P_2 = (-1, 2, -4)$$

2. (4pts.) Give the equations of the cross-sections of the function  $f(x, y) = x^2 + 1$  when  $x = 1$  and when  $y = -1$ .

$f(x, y) = x^2 + 1$  is a parabolic cylinder, whose equation is independent of  $y$ . Therefore, the equations of the two cross-sections are

$$x = 1: f(1, y) = 2 \quad \text{and} \quad y = -1: f(x, -1) = x^2 + 1$$

Note that  $f(1, y) = 2$  is the horizontal line  $z = 2$  in the  $x = 1$  plane, and that the  $y = \text{const.}$  cross-sections have all the same equation  $z = x^2 + 1$ .