Tools 1
MA1032 Data, Functions & Graphs

Sidney Butler

Michigan Technological University

September 5, 2006
Definition

A linear equation in two variables, say the variables $x$ and $y$, is an equation that can be written in the form $ax + by = c$ where $a$, $b$, and $c$ are constants and $a$ and $b$ are not both zero.

Example

Determine if the following equations are linear.

1. $3x - (2 - 4y) = x - y + 1$
2. $\frac{x+2}{3} - y = \frac{y}{5}$
3. $x^2 - (x - 3)^2 = 3y$
Solving Exactly vs. Solving Approximately

Example

Solution: $\frac{3\pi}{\sqrt{2}}$ or 6.66432440724

Each has its benefits.
Definition

A **system** of equations is a group of equations.

Definition

The **simultaneous** solution to a system of equations is a solution that satisfies all of the equations in the system.
Example

Show that the coordinate \((4, -1)\) is the simultaneous solution to the following system of equations.

\[
\begin{align*}
x + y &= 3 \\
x - y &= 5
\end{align*}
\]

Example

Is the coordinate \((1, 2)\) the simultaneous solution to the following system of equations?

\[
\begin{align*}
3x - 2y &= 6 \\
y &= 2x - 5
\end{align*}
\]
Methods for solving systems of equations.

1. **Substitution.**
   Solve for one variable in an equation and then plug it into the others.

2. **Elimination.**
   Multiply one equation by a convenient constant and then add the equation to another equation.
Example

Solve the following systems of equations.

1. \[
\begin{align*}
2x - y &= 10 \\
x + 2y &= 15
\end{align*}
\]

2. \[
\begin{align*}
x &= 7y - 9 \\
4x - 15y &= 26
\end{align*}
\]

3. \[
\begin{align*}
3x - y &= 17 \\
-2x - 3y &= -4
\end{align*}
\]

4. \[
\begin{align*}
2x + 3y &= 7 \\
y &= -\frac{3}{5}x + 6
\end{align*}
\]
Application.
Finding the intersection of two lines.

Example
Find the intersection of the lines $y = x + 1$ and $2x + 3y = 12$. 
Example

Where do the lines $y = 2x - 3.5$ and $y = -\frac{1}{2}x + 4$ intersect?
Summary.

1. Linear Equations
2. Exact vs. Approximate Solutions
3. Systems of Equations
4. Substitution & Elimination