

## • Solution of Section 5.1–5.2

### Section 5.1

#1. Each drawn card can be thought as a Bernoulli trial with "success"="this card is a spade". So,

$$p = P(\text{success}) = 1/4.$$

Let  $X$ ="the number of the success in 8 Bernoulli trials, then  $X \sim B(8, p)$ . The probability we need to calculate is

$$P(X = 4) = \binom{8}{4} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^4 = 0.087.$$

#3. We can consider the birth day of each of the six students as a Bernoulli trial with "success"="the birth day either in April or October". So,

$$p = P(\text{success}) = 1/6.$$

Let  $X$ ="the number of success among the six students", then  $X \sim B(6, p)$ . The probability we need to calculate is

$$P(X = 3) = \binom{6}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^3 = 0.054.$$

#6. Assume that the manufacturer's claim is true i.e. the defective rate of the nails is 3%. We calculate the event  $A$ =(two defectives among the 24 randomly selected sample) [note that this event has occurred]. If the probability of event  $A$  is very small (usually  $\leq 5\%$ ), we will reject the assumption i.e. the manufacturer's claim is not true. Let  $X$  denote the number of defective nails. If the assumption is true, we have

$$\begin{aligned} P(A) &= P(X = 2) \\ &= \binom{24}{2} (0.03)^2 (0.97)^{22} \\ &= 0.127 \end{aligned}$$

we can not reject the manufacturer's claim.

We can also consider the event  $A = \{\text{at least two defectives among the 24 random selected samples}\}$ . In this case

$$\begin{aligned} P(A) &= P(X \geq 2) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \binom{24}{0} (0.03)^0 (0.97)^{24} - \binom{24}{1} (0.03)^1 (0.97)^{23} = 0.162 \end{aligned}$$

We also can not reject the manufacturer's claim.

#7. Let  $p$  and  $q$  denote the probability function of  $X$  and  $Y$ , respectively.

Then,

$$p(x) = \binom{4}{x} (0,6)^x (0,4)^{4-x}; \quad x=0,1,2,3,4;$$

$$\begin{aligned} q(y) &= P(Y = y) \\ &= P(X = \frac{y-1}{2}) \\ &= \binom{4}{\frac{y-1}{2}} (0,6)^{(y-1)/2} (0,4)^{4-(y-1)/2}; \quad y=1,3,5,7,9; \end{aligned}$$

#13. We can consider every day's price as a Bernoulli trial with "success" = "the price moves up", so

$$p = P(\text{success}) = \frac{1}{3}.$$

Let  $X$  denote the number of success i.e. the number of the days in which the price will moves up, Then, the probability we need to calculate is

$$P(X = 3) = \binom{6}{3} (1/3)^3 (2/3)^3.$$

**Section 5.2.**

#1. Let  $X$  denote the number (in a week) of the tickets that win. Then  $X$  is a Binomial r.v. with parameter  $(60, 5\%)$ . So, we can consider approximately  $X$  is Poisson r.v. with parameter  $\lambda = np = 60 \times 0,05 = 3$ . Using this Poisson approximation, the probability that he win next week (he has at least one ticket that win)

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \frac{\lambda^0}{0!} e^{-\lambda} \\ &= 1 - e^{-3} = 0.9502. \end{aligned}$$

#5. Let  $X$  denote the number of vacant rooms in a day. Then,  $X$  approximately is Poisson with parameter  $\lambda = 30$ . thus, the desired probability is

$$1 - \sum_{i=0}^{29} \frac{(30)^i}{i!} e^{-30} = 0.823.$$

#7. From the equation

$$P(X = 1) = P(X = 3)$$

we can get

$$\frac{\lambda^1}{1!} e^{-\lambda} = \frac{\lambda^3}{3!} e^{-\lambda},$$

from which we can get  $\lambda = \sqrt{6}$ . The answer is

$$P(X = 5) = \frac{(\sqrt{6})^5}{5!} e^{-\sqrt{6}} = 0.063.$$

#12. Let  $N(t)$  denote the number of earthquake in  $t$  weeks.  $N(t)$  is Poisson process i.e.  $P(N(t) = k) = \frac{(t\lambda)^k}{k!} e^{-t\lambda}$ . The probability that the next earthquake occurs after weeks is

$$P(N(2) = 0) = e^{-2\lambda} = e^{-6} = 0.00248.$$