• Solution of Section 4.2–4.5

Section 4.2:

1.

Let X denote the absolute value of the difference of the outcomes. The possible value of X is $A = \{0, 1, 2, 3, 4, 5\}$

the probability associated with the values are

X	0	1	2	3	4	5
P(X=i)	6/36	10/36	8/36	6/36	4/36	2/36

3. Here, $X = 0$ means no one has disease in the population. $X = k$ $(K \geq 1)$ means that test the person one-by-one and the first $k-1$ persons have no disease and the kth person has disease. So, the possible values of $X = \{0, 1, 2, \ldots, N\}$ and

$$P(X = k) = \begin{cases} (1-p)^{k-1}p; \ 1 \leq k \leq N \\ (1-p)^N; \ \text{k=0} \end{cases}$$

#5. (b)

$$\begin{aligned}
P(X &< 1) = F(1-) = 1/2; \\
P(X &= 1) = F(1) - F(1-) = 1/6 \\
P(1 &\leq X < 2) = F(2-) - F(1-) = 1/4; \\
P(X &> 2) = 1 - P(X \leq 2) = 1 - F(2) = 1 - 1/2 = 1/2; \\
P(X &= 3/2) = F(3/2) - F(3/2-) = 0; \\
P(1 &< X \leq 6) = P(X \leq 6) - P(X \leq 1) = F(6) - F(1) = 1 - 2/3 = 1/3.
\end{aligned}$$

#7.(a) Using F(6-)=1; we can get $k(-36 + 72 - 3) = 1; sok = 1/33$.

(b). P$(2 \leq X \leq 4) = F(4) - F(2-) = 29/33 - 4/33 = 25/33$.

(c) P(X¿3)=1-P(X$\leq$ 3$) = 1 - F(3) = 1 - 24/33 = 9/33$.

(d)

$$P(X \le 4 | X \ge 3) = \frac{P(3 \le X \le 4)}{P(X \ge 3)} = \frac{F(4) - F(3-)}{1 - F(3-)} = \frac{29/33 - 9/33}{1 - 9/33} = 5/6.$$

#9. (a)

$$\begin{aligned} P(|X| &\le t) = P(-t \le X \le t) = P(X \le t) = P(X \le t) - P(X < -t) \\ &= F(t) - [1 - P(X \ge -t)] = F(t) - [1 - P(X \le t)] = 2F(t) - 1. \end{aligned}$$

(b) Using part (a), we have

$$P(|X| > t) = 1 - P(|X| \le t) = 1 - [2F(t) - 1] = 2[1 - F(t)].$$

(c)

$$\begin{aligned} P(X &= t) = 1 + P(X = t) - 1 = P(X \le t) + P(X > t) + P(X = t) - 1 \\ &= P(X \le t) + P(X \ge t) - 1 = P(X \le t) + P(X \le -t) - 1 \\ &= F(t) + F(-t) - 1. \end{aligned}$$

Section 4.3.

#1. The distribution function of X see the answer in the book.

#3. The possible value of X are $2, 3, 4, \ldots, 12$. Thus

$$\begin{aligned} P(2) &= P(X = 2) = P(\{1, 1\}) = 1/36; \\ P(3) &= P(X = 3) = P(\{(1, 2), (2, 1)\} = 2/36; \\ P(4) &= P(X = 4) = P(\{(1, 3), (2, 2), (3, 1)\}) = 3/36; \end{aligned}$$

Similarly,

X	5	6	7	8	9	10	11	12
$p(i)$	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

#4. Let p be the probability of X. We have

x	-2	2	4	6
$p(x)$	1/2	1/10	13/45	1/9

#7. (a)

$$1 = \sum_{x=1}^{5} kx \Longrightarrow k = 1/15$$

(b)

$$k(-1)^2 + k + 4k + 9k = 1 \Longrightarrow k = 1/15$$

(c)

$$1 = \sum_{x=1}^{\infty} k(\frac{1}{9})^x = k\frac{1/9}{1-1/9} = k/8 \Longrightarrow k = 8.$$

(d)

$$1 = k(1+2+\cdots n) = k\frac{n(n+1)}{2} \Longrightarrow k = \frac{2}{n(n+1)}$$

(e)

$$1 = k(1^2 + 2^2 + \cdots n^2) = k\frac{n(n+1)(2n+1)}{6} \Longrightarrow k = \frac{6}{n(n+1)(2n+1)}$$

#9. For $x < 0, F(x) = 0$. If $x \geq 0$, for some nonnegative integer n, $n \leq x < n+1$, and we have that

$$\begin{aligned} F(x) &= \sum_{i=0}^{n} \frac{3}{4}(\frac{1}{4})^i = \frac{3}{4}[1 + (\frac{1}{4}) + (\frac{1}{4})^2 + \cdots + (\frac{1}{4})^n] \\ &= \frac{3}{4}\frac{1-(\frac{1}{4})^{n+1}}{1-\frac{1}{4}} = 1 - (\frac{1}{4})^{n+1}. \end{aligned}$$

Thus

$$F(x) = \begin{cases} 0 \text{ if } X < 0 \\ 1 - (\frac{1}{4})^{n+1} \text{ if } n \leq x < n+1, n = 0, 1, 2, \ldots. \end{cases}$$

Section 4.4

#1 Let X be the fine that the citizen pays on a random day. Then

$$E(X) = 25(0.60) + 0(0.40) = 15.$$

Therefore, it is much better to park legally.

#6. Let X denote the number of defective fuses among the three selected fuses. Then the possible value of X is A={0,1,2,3} and

$$P(X=i)=\frac{\begin{pmatrix}5\\ i\end{pmatrix}\begin{pmatrix}15\\ 5-i\end{pmatrix}}{\begin{pmatrix}20\\ 3\end{pmatrix}}.$$

So,

$$E(X)=\sum_{i=0}^{3} iP(X=i)=0.75.$$

#9. (a) Let A={-2,-1,0,1,2}. For $x\in A$, $p(x)=(|x|+1)^2/27>0$. For $x\notin A$, let $p(x)=0$. Further

$$\sum_{i=-2}^{2} p(x)=\frac{9}{27}+\frac{4}{27}+\frac{1}{27}+\frac{4}{27}+\frac{9}{27}=1.$$

So, $p(x)$ is a probability of a random variable.

(b).

$$\begin{aligned}
E(x) &= \sum_{i=-2}^{2} xp(x)=0\\
E(|x|) &= \sum_{i=-2}^{2} |x|p(x)=\frac{44}{27}\\
E(X^2) &= \sum_{i=-2}^{2} x^2p(x)=\frac{80}{27}\\
E(2X^2-5X+7) &= 2(\frac{80}{27})-5(0)+7=\frac{349}{27}
\end{aligned}$$

#11. The probability function of X is give by

x	-3	0	3	4
$p(x)$	3/8	1/8	1/4	1/4

Hence,

$$\begin{aligned}
E(X) &= -3\cdot\frac{3}{8}+0\cdot\frac{1}{8}+3\cdot\frac{1}{4}+4\cdot\frac{1}{4}=\frac{5}{8},\\
E(X^2) &= 9\cdot\frac{3}{8}+0\cdot\frac{1}{8}+9\cdot\frac{1}{4}+16\cdot\frac{1}{4}=\frac{77}{8},\\
E(|X|) &= 3\cdot\frac{3}{8}+0\cdot\frac{1}{8}+3\cdot\frac{1}{4}+4\cdot\frac{1}{4}=\frac{23}{8},\\
E(X^2-2|x|) &= \frac{77}{8}-2\cdot\frac{23}{8}=\frac{31}{8}
\end{aligned}$$

#12.

$$\begin{aligned}
E(X) &= \sum_{i=1}^{10} i\frac{1}{10}=\frac{11}{2}\\
E(X^2) &= \sum_{i=1}^{10} i^2\frac{1}{10}=\frac{77}{2},\ \text{so}\\
E[X(11-X)] &= 11E(X)-E(X^2)=11\cdot\frac{11}{2}-\frac{77}{2}=22.
\end{aligned}$$

Section 4.5

#1. On average , in the long run, the two businesses have the same profit. The one that has a profit with lower standard deviation should be chosen by Mr. Jones because he is interested in steady income. Therefore he should chose the first business.

#3.

$$\begin{aligned}
E(X) &= \sum_{x=-3}^{3} xp(x)=-1\\
E(X^2) &= \sum_{x=-3}^{3} x^2p(x)=4.
\end{aligned}$$

Therefore $Var(X)=4-1=3$.

#7. $E(X^2-2X)=3$ implies that $E(X^2)-2E(X)=3$. Substituting $E(X)=1$ in this relation gives $E(X^2)=5$. Hence,

$$Var(X)=E(X^2)-[EX]^2=5-1=4.$$

So,

$$Var(-3X+5) = 9Var(X) = 9 \times 4 = 36.$$