Solution 5th week's homework

- Section 3.1

- #1.

$$P(W|U) = \frac{P(WU)}{P(U)} = \frac{0.15}{0.25} = 0.6.$$

- #3.

$$\frac{0.20}{0.32} = 0.625.$$

- #5.

$$\frac{30-20}{30-15} = \frac{2}{3}.$$

- # 9. Let A={ exact two are trout}, B={at least three are carp}={at most 5 are trouts}. The probability we want to calculate is

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\frac{\binom{40}{2}\binom{65}{6}}{\binom{105}{8}}}{1 - \sum_{i=0}^{2} \frac{\binom{40}{8-i}\binom{65}{i}}{\binom{105}{8}}}.$$

Section 3.2

- # 2. Let A_i = {the ith resume is not sent to a marketing in Maryland} ($i = 1, 2, \ldots, 6$). The probability we need to calculate is

$$\begin{aligned} P(A_1A_2\cdots A_6) &= P(A_1)P(A_2|A_1)P(A_3|A_1A_2)\cdots P(A_6|A_1A_2A_3A_4A_5) \\ &= \frac{11}{14}\cdot\frac{10}{13}\cdot\frac{9}{12}\cdot\frac{8}{11}\cdot\frac{7}{10}\cdot\frac{6}{9} = 0.15. \end{aligned}$$

- # 4. Let $D_i = \{\text{the } i\text{th one is defective}\}$.

  (a). The probability we need to calculate is

$$\begin{aligned} P(D_1D_2D_3D_4) &= P(D_1)P(D_2|D_1)P(D_3|D_1D_2)P(D_4|D_1D_2D_3) \\ &= \frac{8}{20}\cdot\frac{7}{19}\cdot\frac{6}{18}\cdot\frac{5}{17} = 0.0144. \end{aligned}$$

  (b). The probability we need to calculate is

$$\begin{aligned} & P(D_1D_2D_3^c) + P(D_1D_2^cD_3) + P(D_1^cD_2D_3) + P(D_1D_2D_3) \\ = \ & \frac{8}{20}\cdot\frac{7}{19}\cdot\frac{12}{18} + \frac{8}{20}\cdot\frac{12}{19}\cdot\frac{7}{18} + \frac{12}{20}\cdot\frac{8}{19}\cdot\frac{7}{18} + \frac{8}{20}\cdot\frac{7}{19}\cdot\frac{6}{18} \\ = \ & 0.344. \end{aligned}$$

Section 3.3

- # 3.

$$\frac{1}{3}(0.75) + \frac{1}{3}(0.68) + \frac{1}{3}(0.47) = 0.633$$

#5.

$$\frac{11}{50} \times \frac{\binom{13}{2}}{\binom{52}{2}} + \frac{12}{52} \times \frac{\binom{13}{1}\binom{39}{1}}{\binom{52}{2}} + \frac{13}{52} \times \frac{\binom{39}{2}}{\binom{52}{2}} = \frac{1}{4}.$$

#9.

$$(0.5)(0.04) + (0.3)(0.02) + (0.2)(0.04) = 0.034$$

#11. The answer is clearly 0.40. This can also be computed from

$$(0.4)(0.75) + (0.4)(0.25) = 0.4$$