• Solution of Section 6.3

Section 6.3

#1. (a) $E(X) = \int_4^\infty \frac{32}{x^2} dx = 8.$

(b) $E(X^2) = \int_4^\infty \frac{32}{x} dx = 32 \ln(x)|_4^\infty = \infty$, so, $Var(X) = E(X^2) - (EX)^2 = \infty$, i.e. does not exist.

#3. The standardized value of the liftime of a car muffler manufactured by company A is

$$\frac{4.25 - 5}{2} = -0.375$$

The corresponding value for company B is

$$\frac{3.75 - 4}{1.5} = -0.167.$$

Therefore, the muffler of company B has performed relatively better.

#4.

$$E(e^X) = \int_0^\infty e^x (3e^{-3x}) dx = \int_0^\infty 3e^{-2x} dx = \frac{3}{2}.$$

#9. The expected value of the length of the other side is given by

$$E(\sqrt{81 - X^2}) = \int_2^4 \sqrt{81 - x^2} \frac{x}{6} dx$$

Let the $u = 81 - x^2$, we get $du = -2xdx$ and

$$\begin{aligned} E(\sqrt{81 - X^2}) &= \int_2^4 \sqrt{81 - x^2} \frac{x}{6} dx \\ &= \frac{1}{12} \int_{65}^{77} \sqrt{u} du = 8.4 \end{aligned}$$

You can also first to calculate the value of $E(X)$

$$E(X) = \int_2^4 \frac{x^2}{6} dx = 3.111$$

and the length of other side is $\sqrt{81 - (EX)^2} = 8.4$.

• Solution of Section 7.1 to 7.3

Section 7.1

#1. $(23-20)/(27-20) = 3/7$.

#3. Let 2:00pm be the original, then a and b satisfy the following system of two equations in two unknown.

$$\begin{cases} \frac{a+b}{2} = 0 \\ \frac{(b-a)^2}{12} = 12. \end{cases}$$

Solving this system, we obtain $a = -6$ and $b = 6$. So, the bus arrives at a random time between 1:54 P.M. and 2:06 P.M..

5. The peobability density function of R, the radius of the sphere is

$$f(r) = \begin{cases} \frac{1}{4-2} = \frac{1}{2} & 2 < r < 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Thus,

$$E(V) = \int_2^4 (\frac{4}{3}\pi r^2)\frac{1}{2}dr = 40\pi.$$

$$P(\frac{4}{3}\pi R^2 < 36\pi) = P(R^3 < 27) = P(R < 3) = \frac{1}{2}.$$

#7. Let X be a random number from $(0, l)$. Then the pdf of X is given by

$$f(x) = \begin{cases} \frac{1}{l} & 0 < x < l \\ 0 & \text{elsewhere.} \end{cases}$$

The probability of desired event is

$$\begin{aligned} P(\min(X, l-X) &\geq \frac{l}{3}) = P(X \geq \frac{l}{3}, l - X \geq \frac{l}{3}) \\ &= P(\frac{l}{3} \leq X \leq \frac{2}{3}l) \\ &= \int_{l/3}^{2l/3} f(x)dx = \frac{1}{3}. \end{aligned}$$

Section 7.2.

1. Since $np = (0.9)(50) = 45$ and $\sqrt{np(1-p)} = 2.12$,

$$\begin{aligned} P(X &\geq 45) = p(X > 44.5) \\ &= P(\frac{X-45}{2.12} \geq \frac{44.5-45}{2.12}) \\ &= P(\frac{X-45}{2.12} \geq -0.24) \\ &= 1 - \Phi(-0.24) = \Phi(0.24) = 0.5948. \end{aligned}$$

9.

$$\begin{aligned} P(74.5 &< X < 75.8) = P(-0.5 < \frac{X-75}{1} < 0.8) \\ &= \Phi(0.8) - \Phi(-0.5) \\ &= \Phi(0.8) - [1 - \Phi(0.5)] \\ &= 0.4796. \end{aligned}$$

11. Let X be the amount of cereal in a box. We want to have

$$P(X \geq 16) \geq 0.9.$$

This gives

$$P(\frac{X-16.5}{\sigma} \geq \frac{16-16.5}{\sigma}) \geq 0.9.$$

or

$$\begin{aligned} 1 - \Phi(-0.5/\sigma) &\geq 0.9 \\ &\Longleftrightarrow \Phi(0.5/\sigma) \geq 0.9 \end{aligned}$$

The smallest value for $0.5/\sigma$ satisfy this inequality is 1.29; so the largest value for σ is obtained from $0.5/\sigma = 1.29$. This gives $\sigma = 0.388$.

15. Let X be the lifetime of a randomly selected light bulb.

$$\begin{aligned} P(X &\geq 900) = P(\frac{X-1000}{100} \geq -1) \\ &= 1 - \Phi(-1) = \Phi(1) = 0.8413. \end{aligned}$$

Hence the company's claim is false.

Section 7.3

1. Let X be the time until the next customer arrives; X is exponential with parameter $\lambda = 3$. Hence $P(X > x) = e^{-\lambda x}$, and $P(X > 3) = e^{-9} = 0.0001234$.

#3. For $-\infty < y < \infty$, the distribution of Y is given by

$$F(y) = P(Y \leq y) = P(-\ln X \leq y) = P(X \geq e^{-y}) = e^{-e^{-y}}.$$

Thus $g(y)$, the probability density function of Y is given by

$$g(y) = F'(y) = e^{-y}e^{-e^{-y}} = e^{-y-e^{-y}}.$$

5. (a) Suppose that the next customer arrives in X minutes. By the memoryless property, the desired probability is

$$P(X < \frac{1}{30}) = 1 - e^{-5/30} = 0.1535.$$

(b). Let Y be the time between the arrival times of the 10th and 11th customers; Y is exponential with $\lambda = 5$. So the answer is

$$P(Y < \frac{1}{30}) = 1 - e^{-5/30} = 0.1535.$$