

# MA5630

## Problem Set 4

Due 21 April 2003

**Instructions:** Solve two of the following problems. Turn in neatly written solutions. For computational problems, the use of MATLAB or *Mathematica* is recommended.

1. Solve the following nonlinear programs and find the Lagrange multiplier corresponding to the solution. You may solve them analytically or numerically. You may use any software for nonlinear equations or unconstrained minimization (e.g. routines from *Mathematica* or MATLAB), but any software for constrained optimization must be written by you.

(a)

$$\begin{aligned} \min \quad & 2(x_1 + 1)^2 + (x_2 + 2)^2 \\ \text{s.t.} \quad & x_1 + x_2 = 1 \\ & \frac{1}{2} \leq x_1 \leq 1. \end{aligned}$$

(b)

$$\begin{aligned} \min \quad & 3(x_1 + 1)^2 + (x_2 - 1)^2 \\ \text{s.t.} \quad & 2x_1^2 \leq x_2 \\ & x_2 \leq 1. \end{aligned}$$

(c)

$$\begin{aligned} \min \quad & 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 1 \\ & x_1^2 + x_2 \leq 0. \end{aligned}$$

2. In a certain experiment, data points  $(t_i, y_i)$ ,  $i = 1, 2, \dots, n$ , are collected. They are expected to satisfy the relationship

$$y_i = (x_1 t_i + x_2) e^{-x_3 t_i} + x_4 \sin(x_5 t_i),$$

where the parameters  $x_1, x_2, x_3, x_4, x_5$  are unknown but satisfy

$$\begin{aligned} x_i &\geq 0, \quad i = 1, 2, 3, 4, 5, \\ x_3 &\leq 1, \\ x_5 &\leq \pi, \\ x_1^2 + x_2^2 + x_3^2 &\leq 1. \end{aligned}$$

The data points are given in the following table:

$t$	0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
$y$	0.70	0.90	0.40	-0.20	0	0.50	0.20	-0.30	-0.20	0.30	0.30

Estimate the parameters by minimizing

$$\frac{1}{2} \sum_{i=1}^n ((x_1 t_i + x_2) e^{-x_3 t_i} + x_4 \sin(x_5 t_i) - y_i)^2$$

subject to the above constraints. (Note: This is a very nonconvex problem, and you will find a poor solution unless your initial estimate of the frequency  $x_5$  is reasonably good. Fortunately, it is easy to estimate  $x_5$  by plotting the data points.)

3. Suppose  $x^*$  is a local minimizer and a nonsingular point of the NLP

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & g(x) = 0, \end{aligned}$$

where  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  and  $g : \mathbb{R}^n \rightarrow \mathbb{R}$  are twice continuously differentiable. Let  $x : [0, \hat{\mu}] \rightarrow \mathbb{R}^n$  be the continuously differentiable function (whose existence was proved in class) with the following properties:

- (a)  $x(\mu)$  is the unique local minimizer of the quadratic penalty function  $Q(\cdot; \mu)$  near  $x^*$ ;
- (b)  $x(0) = x^*$ .

Prove that there exists a constant  $M > 0$  such that

$$\|x(\mu) - x^*\| \leq \mu M.$$