MA 4310: Abstact Algebra Homework Solutions

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AATA: Chapter 1 exercises

1 Suppose that

$$\begin{aligned} A &= \{x : x \in \mathbb{N} \text{ and } x \text{ is even}\}, \\ B &= \{x : x \in \mathbb{N} \text{ and } x \text{ is prime}\}, \\ C &= \{x : x \in \mathbb{N} \text{ and } x \text{ is a multiple of } 5\} \end{aligned}$$

Describe each of the following sets.

(a) $A \cap B$

(b) $A \cap C$

(c) $A \cup B$

(d) $A \cap (B \cup C)$

Solution.

(a) $A \cap B = \{2\}$ the set of all natural numbers that are both even and prime.

(b) $B \cap C = \{5\}$ the set of all primes that are multiples of 5.

(c) $A \cup B$ is the set of all natural numbers that are either even or prime.

(d) $A \cap (B \cup C) = \{2\}$. Because 2 is the only number that is both even and either a prime or multiple of 5.

□

8 Prove $A \subset B$ if and only if $A \cap B = A$.

Proof. Suppose $A \subset B$ and let $a \in A$. Then $a \in b$ and thus $a \in A \cap B$. Hence $A \subset A \cap B$. Therefore $A \cap B = A$, because by definition $A \cap B \subset A$.

Conversly suppose $A \cap B = A$. By definition $A \cap B \subset B$. Hence $A = A \cap B \subset B$. □

GTN: Chapter 1 exercises