REVIEW 2 KEY

1. A surface is given by equation \(xyz + x + y + e^z = 2\). Find an equation of the tangent plane to the surface at point \((2, -1, 0)\).
   Answer: \(x + y - z - 1 = 0\).

2. Given \(f(x, y) = e^{x^2 + y^2}\), \(x = u \sin v\), and \(y = u \cos v\), use a chain rule to find \(\frac{\partial f}{\partial u}\) and \(\frac{\partial f}{\partial v}\).
   Answer: \(2ue^{u^2}\) and 0.

3. Find all the second partial derivatives of the function \(f(x, y) = \sqrt{x^2 - y^5}\).
   Answer: \(f_{xx} = -\frac{y^5}{(x^2 - y^5)^{3/2}}\), \(f_{xy} = \frac{5xy^4}{2(x^2 - y^5)^{3/2}}\), \(f_{yy} = \frac{5(3y^8 - 8x^2y^3)}{4(x^2 - y^5)^{3/2}}\).

4. Find all the critical points of the function \(f(x, y) = xy e^{x^2 - 2y}\) and classify them as local maxima, minima, saddle point(s), or none of these.
   Answer: \((0, 0)\), a saddle point; \((-\frac{1}{2}, \frac{1}{2})\), a local minimum point.

5. Find the largest and smallest values of \(2x^3 + 4y^2\) subject to the constraint \(x^2 + 4y^2 \leq 4\).
   Answer: largest: 16, at \((2, 0)\); smallest: \(-16\), at \((-2, 0)\).

6. What is the sign of the integral \(\int_{R} (y^2 - 2y - x) \, dA\) over the region \(R\) on the \(xy\)-plane defined by the inequalities \(0 \leq x \leq 1\) and \(0 \leq y \leq 2\)? Explain.
   Solution:
   If \(0 < x < 1\) and \(0 < y < 2\), then the integrand \(y^2 - 2y - x < y^2 - 2y = y(y - 2) < 0\), so the integral is negative.