REVIEW 2 KEY

1. A surface is given by equation $xyz + x + y + e^z = 2$. Find an equation of the tangent plane to the surface at point $(2, -1, 0)$.

Answer: $x + y - z - 1 = 0$.

2. Given $f(x, y) = e^{x^2+y^2}$, $x = u \sin v$, and $y = u \cos v$, use a chain rule to find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$.

Answer: $2ue^{u^2}$ and 0.

3. Find all the second partial derivatives of the function $f(x, y) = \sqrt{x^2 - y^5}$.

Answer: $f_{xx} = -\dfrac{y^5}{(x^2 - y^5)^{3/2}}$, $f_{xy} = \dfrac{5xy^4}{2(x^2 - y^5)^{3/2}}$, $f_{yy} = \dfrac{5(3y^8 - 8x^2y^3)}{4(x^2 - y^5)^{3/2}}$.

4. Find all the critical points of the function $f(x, y) = xye^{x-2y}$ and classify them as local maxima, minima, saddle point(s), or none of these.

Answer: $(0, 0)$, a saddle point; $(-1, \frac{1}{2})$, a local minimum point.

5. Find the largest and smallest values of $2x^3 + 4y^2$ subject to the constraint $x^2 + 4y^2 \leqslant 4$.

Answer: largest: 16, at $(2, 0)$; smallest: -16, at $(-2, 0)$.

6. Find the integral $\int_R (2y - x)\, dA$ over the region R on the xy-plane defined by the inequalities $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 2 - x$.

Answer: 5/3.