

REVIEW 1: Key

1. Find the distances of the point (2, 3, -1) to

(a) the  $xy$ -plane

(b) the point (1, 4, -3)

Key: (a)  $|-1| = 1$ ; (b)  $\sqrt{(1-2)^2 + (4-3)^2 + (-3-(-1))^2} = \sqrt{6}$ .

2. Give an equation of the sphere of radius 2 centered at the point (1, -2, 3).

Key:  $(x-1)^2 + (y+2)^2 + (z-3)^2 = 2^2$ , or  $x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$ .

3. Depict the plane whose equation is  $y = 2$ .

Key: This is the plane parallel to the  $xz$ -plane that crosses the  $y$ -axis at point  $y = 2$ .

4. Depict the surfaces whose equations are

(a)  $z = y^2$

(b)  $z = 3 - x - 2y$

Key:

(a) This is the parabolic cylinder that can be obtained by moving the standard parabola  $z = y^2$  (in the  $yz$ -plane, with axis  $z$ ) along the  $x$ -axis.

(b) This is the plane whose  $x$ -,  $y$ -, and  $z$ -intercepts are 3, 1.5, and 3, respectively. So, you have three points on this plane. Now you can show the triangle with these three points as its vertices.

5. Match the functions (i)  $z = 2x + y - 1$ ; (ii)  $z = 1/(x^2 + y^2)$ ; (iii)  $z = 1 - e^{-(x^2+4y^2)}$ ; (iv)  $z = 5 - x^2$  with

(a) the cross-sections at  $x = 0, 1, 2$

(b) the contour diagrams

Key:

(a)

(i)  $\leftrightarrow$  lower-right

(ii)  $\leftrightarrow$  upper-right

(iii)  $\leftrightarrow$  upper-left

(iv)  $\leftrightarrow$  lower-left

(b)

(i)  $\leftrightarrow$  lower-right

(ii)  $\leftrightarrow$  upper-right

(iii)  $\leftrightarrow$  lower-left

(iv)  $\leftrightarrow$  upper-left

6. What is the linear function of  $x$  and  $y$  with value  $z = 2$  when  $x = 1$  and  $y = -1$ , slope 3 in the  $x$  direction and slope  $(-0.5)$  in the  $y$  direction?

Key:  $f(x, y) = 2 + 3(x - 1) - 0.5(y + 1)$ , or  $f(x, y) = 3x - 0.5y - 1.5$ .

7. Match the functions  $f(x, y, z) = x^2 + y^2$  and  $g(x, y, z) = x^2 + y^2 + z^2$  with the following families of level surfaces.

Key:  $f \leftrightarrow$  right;  $g \leftrightarrow$  left.

**8.** Given the function  $f(x, y) = 3x^2 - y^3$  and the point  $(1, 2)$ ,

(a) use difference quotients with  $\Delta x = 0.1$  and  $\Delta y = 0.1$  to estimate the partial derivatives  $f_x(1, 2)$  and  $f_y(1, 2)$ ;

(b) find the same partial derivatives exactly.

Key:

(a)  $f_x(1, 2) \approx \frac{f(1.1, 2) - f(1, 2)}{0.1} = 6.3$ ;  $f_y(1, 2) \approx \frac{f(1, 2.1) - f(1, 2)}{0.1} = -12.61$ .

(b)  $f_x = 6x = 6$ ;  $f_y = -3y^2 = -12$ .

**9.** Given the function  $f(x, y) = xe^{-2y}$  and the point  $(1, 0)$ ,

(a) find the differential at this point

(b) write down the equation of the tangent plane to the graph of the function at the given point

(c) compute the linear approximation to the value of the function at point  $(1.01, -0.02)$

Key:

(a)  $f_x = e^{-2y} = 1$ ,  $f_y = -2xe^{-2y} = -2$ ,  $df = dx - 2dy$ .

(b)  $z = 1 + (x - 1) - 2(y - 0)$ , or  $z = x - 2y$ .

(c)  $f(1.01, -0.02) \approx 1 + (1.01 - 1) - 2(-0.02 - 0) = 1.05$ .

**10.** Given the function  $f(x, y) = \ln(1/x - \sin y)$  and the point  $(1, 0)$ ,

(a) find the gradient at this point

(b) the directional derivative at this point in the direction of the vector  $\vec{v} = 2\vec{i} + 3\vec{j}$ .

Key:

(a)  $f_x = \frac{-1/x^2}{1/x - \sin y} = -1$ ,  $f_y = \frac{-\cos y}{1/x - \sin y} = -1$ ,  $\text{grad } f = (-1, -1) = -\vec{i} - \vec{j}$ .

(b)  $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{2}{\sqrt{13}}\vec{i} + \frac{3}{\sqrt{13}}\vec{j}$ ,  $f_{\vec{u}} = \vec{u} \cdot \text{grad } f = -\frac{2}{\sqrt{13}} - \frac{3}{\sqrt{13}} = -\frac{5}{\sqrt{13}}$ .