FINAL REVIEW

Sections 16.5, 17.1–3, 18.1–4, 19.1

1. Given the force field $\vec{F} = \vec{i} + 2\vec{j} - t\,\vec{k}$, find the work over the curve C given by the parametric equations

$$\begin{cases} x = 3\cos\sqrt{t} \\ y = 3\sin\sqrt{t} \\ z = 4\sqrt{t} \end{cases}$$

of a curve C, where $0 \leqslant t \leqslant \pi^2$.

2. For each of the following force fields that are path-independent, find the corresponding potential function:

(a) $\vec{F} = -y\,\vec{i} + x\,\vec{j}$; (b) $\vec{F} = (x^3 - 3xy^2, y^3 - 3x^2y)$.

3. For each of the force fields in Problem 2 that are not path-independent, find the work done over the circle of radius 2 centered at point $(0, 1)$, traced out clockwise. Do this in two ways: (i) using Green's theorem and (ii) without using it.

4. Find the following fluxes:

(a) of the field $\vec{F} = xyz\,\vec{i} + x\,\vec{j} - y\,\vec{k}$ through the square of side 2 centered on the x-axis in plane $x = 3$ and oriented in the negative x-direction;

(b) of the field $\vec{F} = -2\vec{r}$ through the sphere of radius 5 centered at the origin and oriented outward.

5. Use the Divergence Theorem to find the flux of the vector field $\vec{F} = (x^3, z, y)$ through the entire boundary surface (oriented outward) of the solid cylinder defined by the inequalities $x^2 + y^2 \leqslant 1$ and $-2 \leqslant z \leqslant 1$.