

REVIEW 3: Sections 16.2–16.5, 17.1–17.2

1. Given the iterated integral $\int_0^3 \int_{y^2}^9 y \sin(x^2) dx dy$, draw the corresponding region of integration on the xy -plane, reverse the order of integration, and evaluate the integral.
2. Find the volume under the graph of the function $f(x, y) = 3xy + y^2$ over the region R defined by the inequalities $x \geq 0$, $y \geq 0$, and $x + 2y \leq 2$.
3. Find the mass of the pyramid bounded by the planes $x = 0$, $y = 0$, $z = 1$, and $3x + 2y + z = 7$ given the mass density function $\delta(x, y, z) = 2z$.
4. Find the double integral

$$\int_{\pi/4}^{3\pi/4} \int_0^{4/\sin \theta} r dr d\theta$$

in polar coordinates and sketch the region of integration. Interpret the double integral geometrically as an area or volume.

5. Using cylindrical coordinates, find the mass of the upper half (above the xy -plane) of the unit ball $x^2 + y^2 + z^2 \leq 1$ given the mass density function $\delta(x, y, z) = z$. Find the same mass using spherical coordinates.
6. Given the parametric equations

$$\begin{cases} x = 3 \cos \sqrt{t} \\ y = 3 \sin \sqrt{t} \\ z = 4\sqrt{t} \end{cases}$$

of a curve C , where $0 \leq t \leq \pi^2$, find the velocity and acceleration vectors as they depend on t . Find also the speed and the length of curve C . Describe the motion and its trajectory in words.