Berry-Esseen bounds for general nonlinear statistics, with applications to Pearson's and non-central Student's and Hotelling's

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Main results

– Initial motivation and general idea
– Application: a BE bound for Pearson's R
– Applications: uniform and nonuniform BE bounds for Student's t and self-normalized sums

Supporting and/or related results

– Comparison: self-normalized sums vs. Student's statistic
– Exact bounds on the closeness between t_p and $t_\infty = N(0,1)$
– Tail monotonicity of t_p
– Exact upper bounds on the mean and exact lower bounds on the exponential moments of the Winsorised-tilted distribution
– An asymptotically Gaussian bound on the Rademacher tails
– Refined and generalized Bennett-Hoeffding bound
– Positive-part moments via the Fourier–Laplace transform
– Improved & generalized von Bahr–Esseen inequality and applications to concentration of measure for separately Lipshitz functions on product spaces
– Optimal re-centering inequality

Initial motivation: Pitman's ARE between Pearson's, Kendall's, and Spearman's correlation coeffs.

- Needed: closeness to normality uniformly near H_0.
- Kendall's and Spearman's coeffs. are U-statistics, with known BE bounds.
- For Pearson's, a BE bound is not found in literature.
  Hardly surprising:
  - an optimal BE bound for Student's t: obtained only in '96, by Bentkus and Götze;
  - A necessary and sufficient condition, in the i.i.d. case, for t to be asymptotically normal: obtained only in '97, by Giné, Götze and Mason.

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Pearson's R: general idea

Let $(Y, Z), (Y_1, Z_1), \dots, (Y_n, Z_n)$ be i.i.d. random points in $\mathbb{R}^2$;
w.l.o.g. $\mathsf{E}\, Y = \mathsf{E}\, Z = 0$ and $\mathsf{E}\, Y^2 = \mathsf{E}\, Z^2 = 1$.

$$\text{Pearson's } R := \frac{\overline{YZ} - \overline{Y}\,\overline{Z}}{\sqrt{\overline{Y^2} - \overline{Y}^2}\sqrt{\overline{Z^2} - \overline{Z}^2}} = f(\overline{V}),$$

where $\overline{V} := \frac{1}{n}\sum_1^n V_i$ and the V_i's are iid copies of
$V := \big(Y, Z, Y^2 - 1, Z^2 - 1, YZ - \rho\big)$,
with $\rho := \mathsf{E}\, YZ = \mathbb{C}\mathrm{orr}(Y, Z)$, so that $\mathsf{E}\, V = 0$.

So, $f(\overline{V}) \approx f(0) + L(\overline{V})$, where $L := f'(0)$ is a linear functional.

From here, using exp. ineqs. for sums in B-spaces by Pinelis–Sakhanenko '85: BE bounds $O(n^{-1/2} \ln^{3/2} n)$ if $\|V\|_3 < \infty$, and $O(n^{1-p/2})$ if $\|V\|_p < \infty$ for some $p \in (2, 3)$.

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The Chen and Shao method '01–'07: a concentration method in Stein-type framework

Given:

- an abstract nonlinear statistic T;
- an abstract linear statistic W.

For $\Delta := T - W$, start with

$$-\mathbb{P}(z-|\Delta| < W \leqslant z) \leqslant \mathbb{P}(T \leqslant z)-\mathbb{P}(W \leqslant z) \leqslant \mathbb{P}(z < W \leqslant z+|\Delta|).$$

A number of applications were given by Chen and Shao.
We modify their method, apply it to $f(\overline{V}) \approx f(0) + L(\overline{V})$, and use other tools to get:
BE-type uniform and nonuniform bounds for statistics of the form $f(\overline{V})$;
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A resulting corollary for Pearson's R:

If e.g. $\rho = \mathsf{E}\, YZ = 0$ and $\tilde\sigma := \sqrt{\mathsf{E}\, Y^2 Z^2} \neq 0$ then

$$\Big|\mathbb{P}\Big(\frac{R}{\tilde\sigma/\sqrt{n}} \leqslant z\Big) - \Phi(z)\Big| \leqslant \frac{4.08}{\sqrt{n}}\big(\|Y\|_6^6 + \|Z\|_6^6\big)\big(1 + \tilde\sigma^{-3}\big).$$

Recall:
here $V = \big(Y, Z, Y^2 - 1, Z^2 - 1, YZ\big)$;
so, $\|V\|_3^3 \asymp \|Y\|_6^6 + \|Z\|_6^6$.
Cf. Bhattacharya–Ghosh '78, Chibisov '80: Asymptotic expansion for the distr. of statistics admitting a stochastic expansion.

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If e.g. $\rho = \mathsf{E}\, YZ = 0$ and $\tilde\sigma := \sqrt{\mathsf{E}\, Y^2 Z^2} \neq 0$ then

$$\Big|\mathbb{P}\Big(\frac{R}{\tilde\sigma/\sqrt{n}} \leqslant z\Big) - \Phi(z)\Big| \leqslant \frac{4.08}{\sqrt{n}}\big(\|Y\|_6^6 + \|Z\|_6^6\big)\big(1+\tilde\sigma^{-3}\big).$$

Recall:
here $V = \big(Y, Z, Y^2 - 1, Z^2 - 1, YZ\big)$;
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Self-normalized sums: Bentkus and Götze, and Shao

Let $Y, Y_1, \dots, Y_n$ be iid with $\mathsf{E}\,Y = 0$ and $\mathsf{E}\,Y^2 = 1$. Let

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$$\begin{aligned}\left|\mathbb{P}(T \leqslant z) - \Phi(z)\right| &\leqslant 25\frac{\mathsf{E}\,|Y|^3}{\sqrt{n}}\,\mathsf{I}\left\{|Y| \leqslant \frac{\sqrt{n}}{2}\right\} + 10.2\,\mathsf{E}\,Y^2\,\mathsf{I}\left\{|Y| > \frac{\sqrt{n}}{2}\right\}\\ &\leqslant 25\frac{\|Y\|_3^3}{\sqrt{n}};\end{aligned}$$

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for $(A_3, A_4, A_6) \in \big\{(1.53, 1.52, 1.34),\ (10.94, 9.40, 11.06 \times 10^{-6})\big\}$;
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Comment: $\max_{z>0} \frac{e^{-z/2}}{1/z^3} \approx 10.75$, attained at $z = 6$.

Cf.: best known nonuniform BE bound for sums of i.i.d. r.v.'s (Michel '81 cum Shevtsova '11): $30.2211 \frac{\|Y\|_3^3}{(|z|^3+1)\sqrt{n}}$.

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	$\omega = 0.1$			$\omega = 0.5$		
	A_3	A_4	A_6	A_3	A_4	A_6
$w_g = 1$	38	36	36	48	48	42
	39	20	7	66	33	13
$w_g = 0$	151	148	147	166	166	165
	170	85	29	229	115	45

Comment: $\max_{z>0} \frac{e^{-z/2}}{1/z^3} \approx 10.75$, attained at $z = 6$.
Cf.: best known nonuniform BE bound for sums of i.i.d. r.v.'s (Michel '81 cum Shevtsova '11): $30.2211 \frac{\|Y\|_3^3}{(|z|^3+1)\sqrt{n}}$.

Self-normalized sums vs. Student's statistic

For all $n > 1$

$$\left| \sup_{z\in\mathbb{R}} \left|\mathbb{P}(t \leqslant z) - \Phi(z)\right| - \sup_{z\in\mathbb{R}} \left|\mathbb{P}(T \leqslant z) - \Phi(z)\right| \right| < \frac{C}{n-1},$$

where t is the Student statistic, T is again the self-normalized sum,

$$C := \left(k - \frac{1}{2}\right) e^{-k} \sqrt{\frac{k}{\pi}} = 0.162\ldots, \quad \text{and} \quad k := 1 + \frac{\sqrt{3}}{2}.$$

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Exact bounds on the closeness between t_p and $t_\infty = N(0,1)$

Let $d_{\mathrm{TV}}(p)$ and $d_{\mathrm{Ko}}(p)$ denote, resp., the total-variation and Kolmogorov distances between t_p and $t_\infty = N(0,1)$. Then

$$\frac{1}{2}\,d_{\mathrm{TV}}(p) = d_{\mathrm{Ko}}(p) < \frac{C}{p} \quad \forall p \in [4,\infty),$$

where

$$C := \frac{1}{4}\sqrt{\frac{7+5\sqrt{2}}{\pi e^{1+\sqrt{2}}}} = \lim_{p\to\infty} p\,d_{\mathrm{Ko}}(p) = 0.158\ldots,$$

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Tail monotonicity of t_p

Let G_p stand for the tail function of t_p. Then

$$0 < p < q \leqslant \infty \implies \frac{G_q(x)}{G_p(x)} \text{ is (strictly) decr. in } x \geqslant 0,$$

whence the stochastic majorization:

$$G_q(x) < G_p(x) \quad \forall x > 0.$$

Note: here the likelihood ratio is not monotone; so, the usual scheme MLR $\implies$ MTR $\implies$ SM doesn't work.

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Exact upper bounds on the mean of the Winsorised-tilted distribution

For each $h > 0$ and each $w \in \mathbb{R}$, the maximum of the tilted mean

$$\frac{\mathsf{E}\, X e^{h(X \wedge w)}}{\mathsf{E}\, e^{h(X \wedge w)}}$$

given $\mathsf{E}\, X$ and $\mathsf{E}\, X^2$ is attained when X has a two-point distribution.

For $\mathsf{E}\, X = 0$ and $w > 0$, this maximum is

$$< \frac{e^{hw} - 1}{w}\, \mathsf{E}\, X^2,$$

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An asymptotically Gaussian bound on the Rademacher tails

$$\mathbb{P}(a_1\varepsilon_1 + \cdots + a_n\varepsilon_n \geqslant x) \leqslant \mathbb{P}(Z > x) + \frac{C\varphi(x)}{9 + x^2}$$
$$< \mathbb{P}(Z > x)\Big(1 + \frac{C}{x}\Big) \quad \forall x > 0,$$

where $\varepsilon_1, \ldots, \varepsilon_n$ are independent Rademacher r.v.'s,

$$a_1^2 + \cdots + a_n^2 = 1,$$

$Z \sim N(0,1)$ with density φ, and
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Refined and generalized Bennett-Hoeffding bound

Let $X_1, \dots, X_n$ be independent r.v.'s, with $S := X_1 + \dots + X_n$.
Take any σ, y, and β in $(0, \infty)$ s.t. $\varepsilon := \frac{\beta}{\sigma^2 y} \in (0,1)$. Suppose that

$$\sum_i \mathsf{E}\, X_i^2 \leqslant \sigma^2, \quad \sum_i \mathsf{E}(X_i)_+^3 \leqslant \beta, \quad \mathsf{E}\, X_i \leqslant 0, \quad \text{and } X_i \leqslant y,$$

for all i. Then

$$\mathsf{E}\, f(S) \leqslant \mathsf{E}\, f(\eta_{\varepsilon,\sigma,y})$$

for all $f \in C^2$ s.t. f and f'' are nondecreasing and convex, where

$$\eta_{\varepsilon,\sigma,y} := \Gamma_{(1-\varepsilon)\sigma^2} + y\tilde{\Pi}_{\varepsilon\sigma^2/y^2},$$

$\Gamma_{a^2} \sim N(0, a^2)$, $\tilde{\Pi}_\theta := \Pi_\theta - \theta$, $\Pi_\theta \sim \text{Poisson}(\theta)$,
and Γ_{a^2} and Π_θ are independent.
Corollary: $\mathbb{P}(S \geqslant x) \leqslant \frac{2e^3}{9}\, \mathbb{P}^{\mathrm{LC}}(\eta_{\varepsilon,\sigma,y} \geqslant x) \quad \forall x \in \mathbb{R}$,
where $\mathbb{R} \ni x \mapsto \mathbb{P}^{\mathrm{LC}}(\eta \geqslant x)$ is the least log-concave majorant of
$\mathbb{R} \ni x \mapsto \mathbb{P}(\eta \geqslant x)$.

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For p and s in $(0,\infty)$, $j \in \overline{-1, \lceil p-1 \rceil}$, and any r.v. X with $\mathsf{E}\, e^{sX} < \infty$ one has

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$$\mathcal{F}_+^{1,2} := \big\{ f \in C^1(\mathbb{R})\colon f(0)=0,\ f \text{ is even,}$$
$$f' \text{ is nondecreasing and concave on } [0,\infty) \big\};$$

e.g., $|\cdot|^p \in \mathcal{F}_+^{1,2} \quad \forall p \in (1,2]$. Then

$$\mathsf{E}\, f(S_n) \leqslant \mathsf{E}\, f(X_1) + C_f \sum_{j=2}^{n} \mathsf{E}\, f(X_j)$$

where $f \in \mathcal{F}_+^{1,2} \setminus \{0\}$, $(S_j)_{j=1}^n$ is a martingale, and the constant $C_f := \sup_{0<x<s<\infty} \frac{1}{f(s)} \big(f(x-s) - f(x) + sf'(x) \big)$ is the best possible, for each $f \in \mathcal{F}_+^{1,2} \setminus \{0\}$; also, $\big\{ C_f\colon f \in \mathcal{F}_+^{1,2} \setminus \{0\} \big\} = [1,2]$, and $C_f = 1$ if $f(x) \equiv x^2$.

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Corollary: concentration of measure for separately Lipshitz (sep-Lip) functions

Let $X_1, \dots, X_n$ be independent r.v.'s with values in $\mathfrak{X}_1, \dots, \mathfrak{X}_n$, resp.; here, all spaces and functions are measurable. Suppose

$$Y := g(X_1, \dots, X_n)$$

and the sep-Lip condition

$$\begin{aligned} &|\operatorname{E} g(x_1, \dots, x_{i-1}, \tilde{x}_i, X_{i+1}, \dots, X_n) \\ &- \operatorname{E} g(x_1, \dots, x_{i-1}, x_i, X_{i+1}, \dots, X_n)| \leqslant \rho_i(\tilde{x}_i, x_i) \end{aligned}$$

holds for some functions $\rho_i \colon \mathfrak{X}_i \times \mathfrak{X}_i \to \mathbb{R}$ and all i, $x_j \in \mathfrak{X}_j$, and $\tilde{x}_i \in \mathfrak{X}_i$.

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Corollary: concentration of measure for sep-Lip functions, continued

Then $\forall f \in \mathcal{F}_+^{1,2} \setminus \{0\} \ \ \forall x_i \in \mathfrak{X}_i$

$$\mathrm{E}\, f(Y) \leqslant f(\mathrm{E}\, Y) + \kappa_f\, C_f \sum_{i=1}^{n} \mathrm{E}\, f\big(\rho_i(X_i, x_i)\big), \qquad (*)$$

where

$$\kappa_f := \sup_{0<a<c<s/2} \frac{c\, f(s-c) + (s-c) f(c)}{c\, f(s-c+a) + (s-c) f(a-c)}.$$

Moreover, $\big\{\kappa_f \colon f \in \mathcal{F}_+^{1,2} \setminus \{0\}\big\} = [1, 2]$.
Also, $\kappa_f = C_f = 1$ if $f(x) \equiv x^2$; for this f,
inequality (*) in the case when $\mathfrak{X}_1 = \cdots = \mathfrak{X}_n$ is a B-space,
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Moreover, $\big\{\kappa_f \colon f \in \mathcal{F}_+^{1,2} \setminus \{0\}\big\} = [1, 2]$.
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Optimal re-centering inequality

Let $\mathcal{F}^{2,3}$ be the class (introduced by Figiel, Hitczenko, Johnson, Schechtman and Zinn '97) of all $f\colon \mathbb{R} \to \mathbb{R}$ such that $f(0) = 0$, f is even and convex, $[0, \infty) \ni t \mapsto f(\sqrt{t})$ is convex, and f'' is concave on $[0, \infty)$.

Then for any $f \in \mathcal{F}^{2,3}$, any zero-mean r.v. X, and any $t \in \mathbb{R}$

$$\mathrm{E}\, f(X) \leqslant c\, \mathrm{E}\, f(X + t),$$

where $c := \frac{17+7\sqrt{7}}{27} = 1.315...$ is the best possible factor.

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