

## Motion in Two Dimensions Teacher's Guide

### Objectives:

1. Use kinematic equations for motion in two dimensions to determine the range of a projectile.
2. Use the equation for torque to determine at what point a balanced ruler will not continue to be in equilibrium.
3. Practice designing procedures for the experimental method.

### Materials:

Each group must have:

1 toy dart gun, 1 meter stick, 1 or more stopwatches

The class must have:

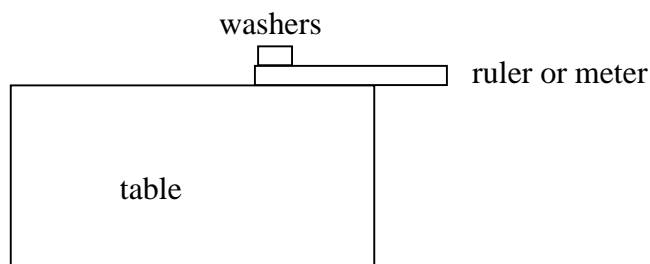
1 ruler or meter stick, up to 20 washers

### Part A:

Part A involves determining maximum range of a toy dart gun. The students begin by finding the muzzle velocity of the gun. They should do this by firing straight up from the table and measuring the time it takes to return to the table. The calculations are outlined in the student handout. The next step is to predict the maximum range of the gun. They can use the projectile motion equations to do this. The maximum range occurs when the gun is fired at a 45 degree angle.

### Part B:

In Part B, have the following setup at the front of the room:



Students will write their own procedure to determine how many washers need to be placed at the hanging end of the ruler before it will fall. Make sure to use washers that are all the same mass. They will use the equation for torque to solve this problem. The torque caused by the washers in this diagram is the weight of the washers times the distance from the pivot point (the end of the table), and is in the counterclockwise direction. Students may point out that additional torque in the clockwise direction is

caused by the mass of the portion of the ruler that is on the table. If so, they can calculate this effect. For instance, if the ruler has 8 inches on the table, then the torque caused by the ruler in the counterclockwise direction is:

$$\tau = \left[ \frac{8in}{12in} \times (\text{Mass of the whole ruler}) \times 9.8 \frac{m}{s^2} \right] \times 4in$$

Everything that is in the [ ] is just the weight of 8 inches of a 12-inch ruler. It is multiplied by 4 inches to get torque, because the center of mass of the 8 inches on the table is 4 inches away from the edge of the table. The total counterclockwise torque is then:

$$\tau = \left\{ \left[ \frac{8in}{12in} \times (\text{Mass of the whole ruler}) \times 9.8 \frac{m}{s^2} \right] \times 4in \right\} + \left\{ (\text{Mass of the washers}) \times 9.8 \frac{m}{s^2} \times 8in \right\}$$

The mass of the ruler has little effect in the end, so if they don't think of this, it should still be okay. Students need to find the number of washers it will take for the ruler to reach equilibrium to get the number of washers it will take to make the ruler fall. This means that the torque in the clockwise direction equals the torque in the counterclockwise direction. The torque in the clockwise direction is found the same way as in the counterclockwise direction, except that the mass of the washers is unknown. Setting the counterclockwise and clockwise torques equal to each other lets them solve for the mass of the washers. To find the number of washers, they just divide the mass of the washers by the mass of one washer. If they get a non-integer number, then they should round up, because they can't have a part of a washer.

Name: \_\_\_\_\_

## Motion in Two Dimensions

### Objectives:

In this lab, you will be using your knowledge of motion in two dimensions to:

1. Predict the range of a toy dart gun.
2. Predict the number of washers required to bring a balancing system out of equilibrium.

### Materials:

Toy dart gun, 2-3 stopwatches, meter stick, 12-in ruler, up to 20 washers, triple beam balance

### Procedure:

#### A. Projectile Motion.

In this part of the lab, you will be using the equations for projectile motion to predict the range of a projectile, the dart from a toy dart gun. To do this, the muzzle velocity (speed at which the dart leaves the gun) must first be calculated.

If the gun is fired at ground level, then the time required to reach the top of the dart's trajectory is equal to half the time it took to reach the floor. Therefore, you will be able to calculate the muzzle velocity using the following equation:

$$y = y_0 + v_0t + \frac{1}{2}at^2$$

where  $t$  is actually  $\frac{1}{2}$  the time it took for the dart to go up and come down ( $t =$  time to go down  $= \frac{1}{2}$  the total time). The acceleration,  $a$  is the acceleration due to gravity. Let  $y_0$  be the unknown maximum height of the trajectory. Then  $y$  is zero, and  $v_0$  is also zero. Remember that the acceleration is negative in this case. It is now possible to determine the maximum height of the trajectory,  $y_0$ .

Knowing  $y_0$ , the muzzle velocity can now be calculated. This time, the equation will represent the upward half of the trajectory. Let the maximum height now be  $y$ . This means that the new  $y_0$  is at zero, and the acceleration is in the opposite direction as  $y$ , so it will be negative.

1. Fire the gun from ground level, and time how long it takes for the dart to reach the ground. Do this at least three times and calculate the average and enter below.

Average total travel time,  $T =$  \_\_\_\_\_ s

2. Divide the time by 2 to get the time to travel either up or down.

$T/2 = t =$  \_\_\_\_\_ s

3. Now use the calculations outlined above to calculate the muzzle velocity of the dart gun.

Muzzle Velocity =  $v_0 =$  \_\_\_\_\_ m/s

- a. Now that you know the muzzle velocity of your gun, you should have all the information necessary to predict the maximum range. HINT: The maximum range occurs for an angle of 45 degrees above the horizontal. Below are some equations that *might* be useful:

$$x = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{0,y}t + \frac{1}{2}a_y t^2$$

$$v_y = v_{0,y} + a_y t$$

Predicted maximum range of gun = \_\_\_\_\_m

5. After calculating your predicted value for the range of the dart gun, ask your teacher to come and watch as you test your predictions. Enter your actual range below and determine the percent error from your predicted value.

Use the following equation to calculate your percent error:

$$\%error = \frac{|predicted - actual|}{actual} \times 100\%$$

Actual maximum range of gun = \_\_\_\_\_m

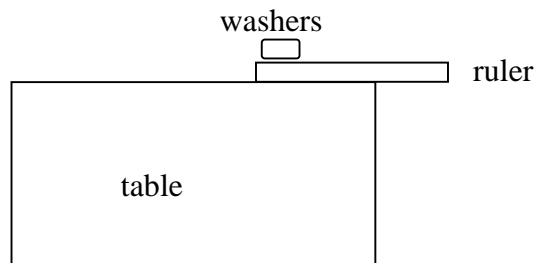
Percent error = \_\_\_\_\_%

Teacher Initials \_\_\_\_\_

6. Is your predicted value close to your actual value? What are some sources of error that may cause your predicted value to be different from the actual value?

## B. Torque

In this part of the lab, you will be predicting the number of washers that must be placed at the end of the ruler at the front of the room that will make the ruler fall. The ruler is placed as in the diagram below.



Your teacher will have placed several washers at the end of the ruler that is on the table. You will be placing your washers on the end of the ruler that is hanging in air. There will also be a triple beam balance next to the ruler so that you can weigh the washers.

1. In the space below, outline the procedure that you will use to predict the number of washers that will move the ruler. HINT: The equation for torque is:

$$\tau = F \times x$$

where  $F$  is the force causing the torque, and  $x$  is the distance from the pivot point at which the force acts.

2. Calculate your predicted number of washers in the space below and enter your predicted number in the space provided.

Predicted Number of Washers = \_\_\_\_\_ washers

3. At the end of the lab period, your teacher will perform the experiment by placing one washer at a time on the end of the ruler until it falls. Record the number of washers it took for the ruler to fall below.

Actual Number of Washers = \_\_\_\_\_ washers

4. How close was your predicted number to the actual number? What might be some causes for error?



