Computer Project 2  
(Part of Final Exam)  

Due: Monday, April 26, 2010

Steady-state heat transfer is governed by the following equation in 2-dimensional planar geometry

$$- \frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) = f(x, y)$$

where $T$ is the temperature (in °C), $k_x$ and $k_y$ are the thermal conductivities in the $x$ and $y$ directions (in W/(cm°C)), and $f$ is the internal heat generation per unit volume (in W/cm$^3$).

Assuming heat transfer is due solely to conduction (i.e. neglecting heat transfer due to convection), the natural boundary condition is

$$k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y = \hat{q}_n$$

where $\hat{q}_n$ is the specified heat flux (in W/cm$^2$). If a boundary is insulated, then $\hat{q}_n = 0$ along that boundary.

Modify your one-dimensional FEM program to solve the following two heat transfer problems on the square domain $\Omega = (0, 1) \times (0, 1)$.

$1$. In each case, take $k = k_x = k_y = 0.3$ W/(cm°C), $f = f_0 = 10$ W/cm$^3$ and $T_0(x, y) = 100^\circ$C.

$2$. Use either linear triangular elements or bilinear rectangular elements. The meshes will be available at http://www.math.mtu.edu/~feigl/courses.html

$3$. Implement a Gauss quadrature rule that will give exact values of all integrals.

$4$. Submit (a) a hard-copy of your program, (b) output tables listing the node numbers, the nodal coordinates $X$, $Y$, and the computed temperatures $T$, and (c) contour plots of your results.